Radiation Pattern and Directivity of an Antenna Array

Introduction

The application calculates the array factor and directivity for a uniform linear antenna array, and then plots the radiation pattern.

```
> restart :
  with( Units[Standard]) :
  with( plots) :
  with( ScientificConstants) :
  with( ColorTools) :
```

Parameters

The number of elements in the uniform array:

> $N := 10$:

Design frequency:

> $f_d := 1\,GHz$:

Permittivity and permeability of free space:

> $\epsilon_o :=$ evalf(Constant(permittivity_of_vacuum, units));

$$8.854187815\,10^{-12}\,\frac{F}{m} \qquad (2.1)$$

> $\mu_o :=$ evalf(Constant(permeability_of_vacuum, units))

$$1.256637062\,10^{-6}\,\frac{H}{m} \qquad (2.2)$$

Phase constant:

> $\beta_o := 2 \cdot \pi \cdot f_d \cdot \sqrt{\mu_o \cdot \epsilon_o}$

$$\beta_o := 20.95845022 \frac{1}{m} \tag{2.3}$$

Wavelength:

> $\lambda_d := 2 \cdot \frac{\pi}{\beta_o}$

$$\lambda_d := 0.2997924580 \text{ m} \tag{2.4}$$

For a maximum at:

> $\phi_m := \frac{\pi}{3.}$:

Inter-element spacing:

> $d := \frac{\lambda_d}{3}$

$$d := 0.09993081933 \text{ m} \tag{2.5}$$

Progressive phase shift between elements:

> $\psi := \beta_o \cdot d \cdot \cos(\phi_m)$

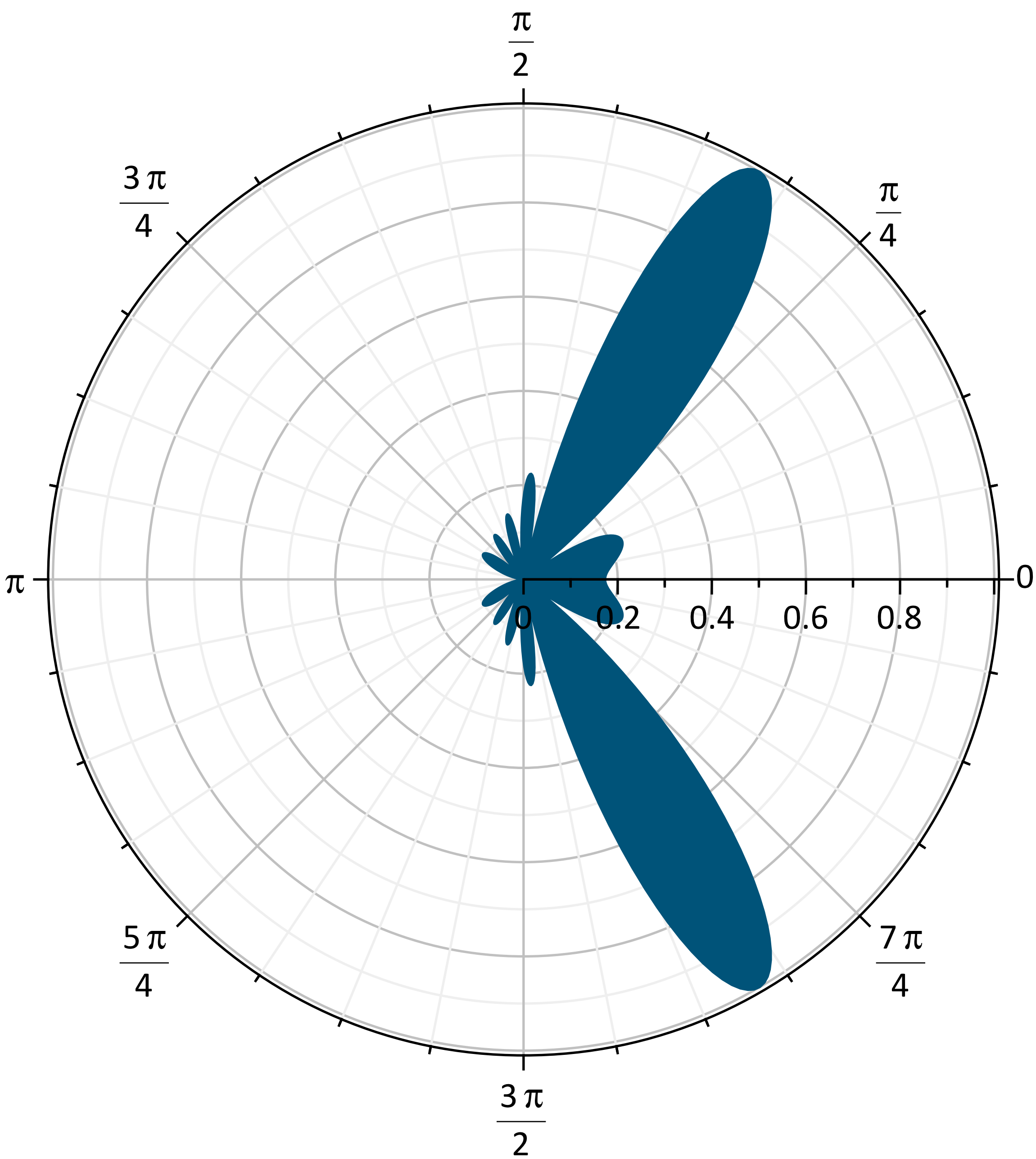
$$\psi := 1.047197551 \tag{2.6}$$

Calculations

Array factor

> $AF := \phi \rightarrow \left| \frac{1}{N} \cdot \frac{\sin\left(\frac{N}{2} \cdot \left(\beta_o \cdot d \cdot \cos(\phi) - \psi\right)\right)}{\sin\left(\frac{1}{2} \cdot \left(\beta_o \cdot d \cdot \cos(\phi) - \psi\right)\right)} \right|$:

> polarplot(AF(ϕ), ϕ = 0 ..2·π, filled, size = [700, 700], color = Color("RGB", [0 / 255, 79 / 244, 121 / 255]), thickness = 0, axesfont = [Calibri], adaptive = false, numpoints = 500, axis = [gridlines = [color = grey]])


π/2
3π/4
π/4
π
0
0
0.2
0.4
0.6
0.8
5π/4
7π/4
3π/2

The directivity for this array is calculated from the total power radiated.

> $P_{tot} := 2 \cdot \mathrm{int}\left(AF(\phi)^2, \phi = 0..2 \cdot Pi, \mathrm{numeric}\right)$

$$P_{tot} := 1.428780370 \quad \textbf{(3.1)}$$

> $D_o := \frac{4 \cdot \pi}{P_{tot}}$

$$D_o := 8.795173059 \quad \textbf{(3.2)}$$

Which, in dB, is:

> $10 \cdot \mathrm{log10}\left(D_o\right)$

$$9.442443893 \quad \textbf{(3.3)}$$

>

Reference

Iskander, Magdi F., Electromagnetic Fields and Waves, Prentice-Hall, Inc., Englewood Cliffs, NJ, 1992.