Application Demonstration

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Extreme Value Analysis of an Electrical Circuit

Introduction

An electrical component, such as a resistor or capacitor, is usually quantified with a nominal value and a tolerance. That is, a resistor could be rated at 5 Ω with a tolerance of 5%; this means the resistance could vary between 4.75 and 5.25 Ω.

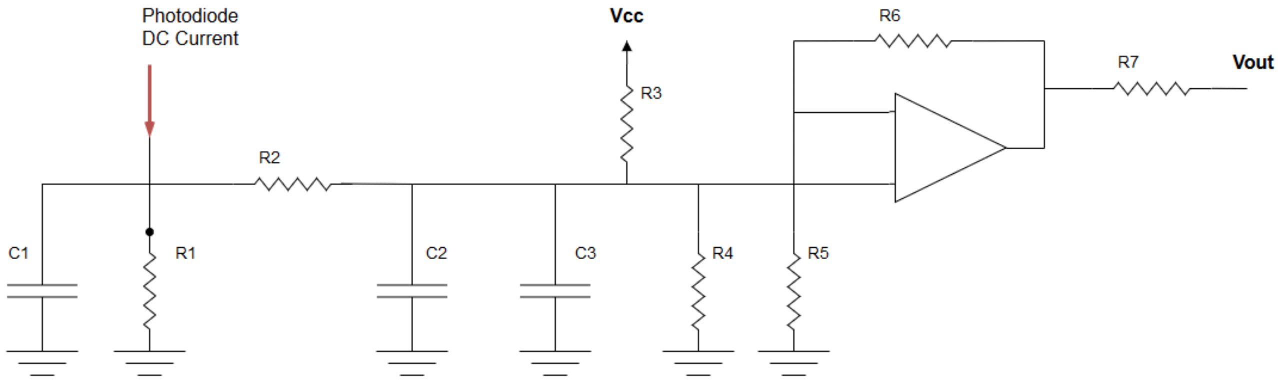
Given the number of components in a circuit and their compounded tolerances, the actual performance of a circuit may not necessarily match its desired performance. This is a source of risk that needs to be managed and mitigated.

Accordingly, electrical engineers need to analyze a circuit over all potential operating conditions.

Extreme Value Analysis (EVA) is a process in which the behavior of a circuit is simulated for every permutation of extreme component parameters - that is, a resistor of 5 Ω ± 5% is simulated at 4.75 Ω and 5.25 Ω, in combination with every permutation of extreme values for all other components (this is a type of worst case circuit analysis).

Given the results of an EVA, a circuit that falls out of spec may have its performance improved by replacing cheaper components that have a loose tolerance with higher quality components that have a tighter tolerance.

This application performs an extreme value analysis of the following circuit (the principles, however, can be extended to any circuit). Light hits a photodiode and generates a current. A non-inverting op-amp then produces a linearly-proportional voltage from the photodiode current. Capacitors are ignored - hence this is a DC analysis.



Component	Value	Units	Tolerance (%)
R1	900	Ω	0.02
R2	67500	Ω	0.02
R3	2050000	Ω	0.01
R4	89200	Ω	0.015
R5	90000	Ω	0.015
R6	87000	Ω	0.005
R7	1.02	Ω	0.07
Vcc	3	V	0.01
P	4.8×10^{-4}	W	0.05

Calculations

```
> restart:
```

Voltage generated by the op-amp

```
> Vout := proc(Rv)
    local R1, R2, R4, R3, R5, R6, R, Vcc, P:
    R1   := Rv[1]:
    R2   := Rv[2]:
    R3   := Rv[3]:
    R4   := Rv[4]:
    R5   := Rv[5]:
    R6   := Rv[6]:
    R    := Rv[7]:
    Vcc  := Rv[8]:
    P    := Rv[9]:
    return (R * P * R1 * (1 / (1 / R4 + 1 / R3)) / ( R1 + R2 + 1
  / (1 / R4 + 1 / R3)) + Vcc / R3 * 1 / (1 / R4 + 1 / R3 + 1 / (R2
  + R1))) * (R5 + R6) / R5;
```

```
end proc:
```

Parameter values and percentage tolerances in the order R1, R2, R3, R4, R5, R6, R, Vcc, P

```
> Rv    := [9000, 67500, 2050000, 89200, 90000, 87000, 1.02, 3,
  0.00048]:
  Delta := [0.02, 0.02,  0.01,    0.015, 0.015, 0.005, 0.07, 0.01,
  0.05]:
```

v is a vector that contains the calculated voltages for every permutation of extreme tolerance values (there are 2^9 = 512 combinations of the two extreme parameter values for each of the nine components in the circuit).

```
> zeta := Vector(512, i -> 1 +~ subs(0 = -1, Bits:-Split(i, bits =
  9)) *~ Delta):

  v := Vector(512, i -> Vout(Rv *~ zeta[i]))
```

$$v := \begin{bmatrix} 4.257792241 \\ 4.039654773 \\ 4.190990302 \\ 4.103357215 \\ 4.257166458 \\ 4.038932838 \\ 4.190326462 \\ 4.159538866 \\ 4.315517230 \\ 4.095319584 \\ \vdots \end{bmatrix} \tag{2.1}$$

512 element Vector[column]

Hence the worst case minimum and maximum voltages are

```
> min(v);
  max(v)
```

$$3.979954450$$

$$5.485606376 \tag{2.2}$$