Application Demonstration

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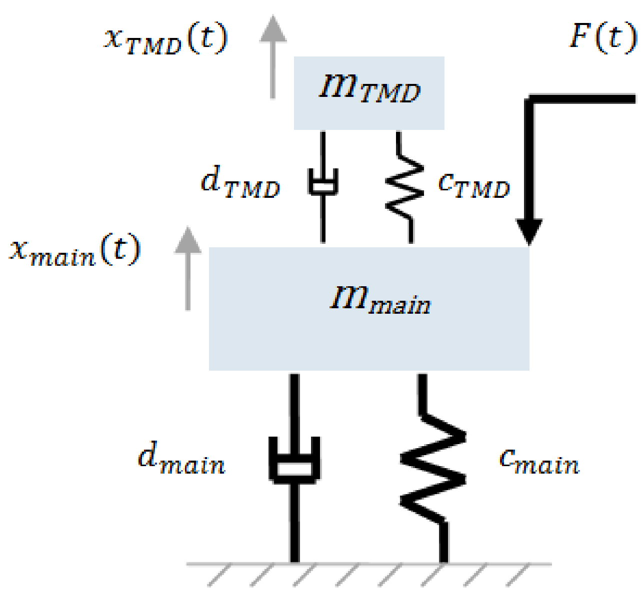
Tuned Mass Damper Design for Attenuating Vibration

Introduction

A mass-spring-damper is disturbed by a force that resonates at the natural frequency of the system. This application calculates the optimum spring and damping constant of a parasitic tuned-mass damper that the minimizes the vibration of the system.

The vibration of system with and without the tuned mass-spring-damper is viewed as a frequency response, time-domain simulation and power spectrum.

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> restart :
  with( DynamicSystems ) :
  with( ColorTools ) :
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Derive Expressions for the Optimum Spring and Damping

Constant of the Tuned Mass Damper

Natural frequency of the tuned mass damper:

$$> \omega_2 := \sqrt{\frac{k_2}{m_2}} :$$

Natural frequency of the main system:

$$> \omega_1 := \sqrt{\frac{k_1}{m_1}} :$$

Ratio of the natural frequencies:

$$> \alpha := \frac{\omega_2}{\omega_1}$$

$$\alpha := \frac{\sqrt{\frac{k_2}{m_2}}}{\sqrt{\frac{k_1}{m_1}}} \tag{2.1}$$

Optimum ratio of natural frequencies:

$$> \alpha_{opt} := \frac{1}{1 + \frac{m_2}{m_1}} :$$

Hence the optimum spring constant of the tuned mass-spring-damper:

$$> k_{2\_opt} := solve\left(\alpha = \alpha_{opt}, k_2\right)$$

$$k_{2\_opt} := \frac{m_1 k_1 m_2}{\left(m_1 + m_2\right)^2} \tag{2.2}$$

Damping ratio:

$$> z := \frac{b_2}{2\, m_2\, \omega_2} :$$

Optimum damping ratio:

> $z_{opt} := \sqrt{\dfrac{3\,\dfrac{m_2}{m_1}}{8\left(1+\dfrac{m_2}{m_1}\right)^3}}$:

Hence the optimum damping constant of the tuned mass-spring-damper:

> $b_{2\_opt} := subs\left(k_2 = k_{2\_opt}, \text{solve}\left(z = z_{opt}, b_2\right)\right)$

$$b_{2\_opt} := \frac{\sqrt{6}\,\sqrt{\dfrac{m_2}{m_1\left(1+\dfrac{m_2}{m_1}\right)^3}}\,\sqrt{\dfrac{m_1 k_1}{\left(m_1+m_2\right)^2}}\,m_2}{2} \tag{2.3}$$

System Parameters

Main spring mass damper parameters:

> $\text{params}_{main} := m_1 = 1.764\,10^5, k_1 = 3.45\,10^7, b_1 = 1.531\,10^5$:

Mass of the tuned mass damper:

> $m_{TMD} := 8165$:

Optimum spring and damping constants of the tuned mass damper are:

> $k_{2\_calc} := \text{eval}\left(k_{2\_opt}, \left[\text{params}_{main}, m_2 = m_{TMD}\right]\right)$;

$$k_{2\_calc} := 1.45873086110^6 \tag{3.1}$$

> $b_{2\_calc} := \text{evalf}\left(\text{eval}\left(b_{2\_opt}, \left[\text{params}_{main}, m_2 = m_{TMD}\right]\right)\right)$;

$$b_{2\_calc} := 26869.77096 \tag{3.2}$$

Parameters for the system with and without a tuned mass damper:

> $\text{params}_{TMD} := \left[\text{params}_{main}, m_2 = m_{TMD}, k_2 = k_{2\_calc}, b_2 = b_{2\_calc}\right]$:
$\text{params}_{noTMD} := \left[\text{params}_{main}, m_2 = 0, k_2 = 0, b_2 = 0\right]$:

Equations of Motion for the Entire System

> $de := m_2 \dfrac{d^2}{dt^2} x_2(t) = -k_2\left(x_2(t) - x_1(t)\right) - b_2\left(\dfrac{d}{dt} x_2(t) - \dfrac{d}{dt} x_1(t)\right), m_1$

$$\frac{d^2}{dt^2} x_1(t) = -k_1 x_1(t) - b_1 \frac{d}{dt} x_1(t) - k_2 (x_1(t) - x_2(t)) - b_2 \left(\frac{d}{dt} x_1(t) - \frac{d}{dt} x_2(t) \right) + F(t) :$$

$ic := x_1(0) = 0, D(x_1)(0) = 0, x_2(0) = 0, D(x_2)(0) = 0 :$

> $sys := DiffEquation([de], [F(t)], [x_1(t)]) :$

Frequency Response

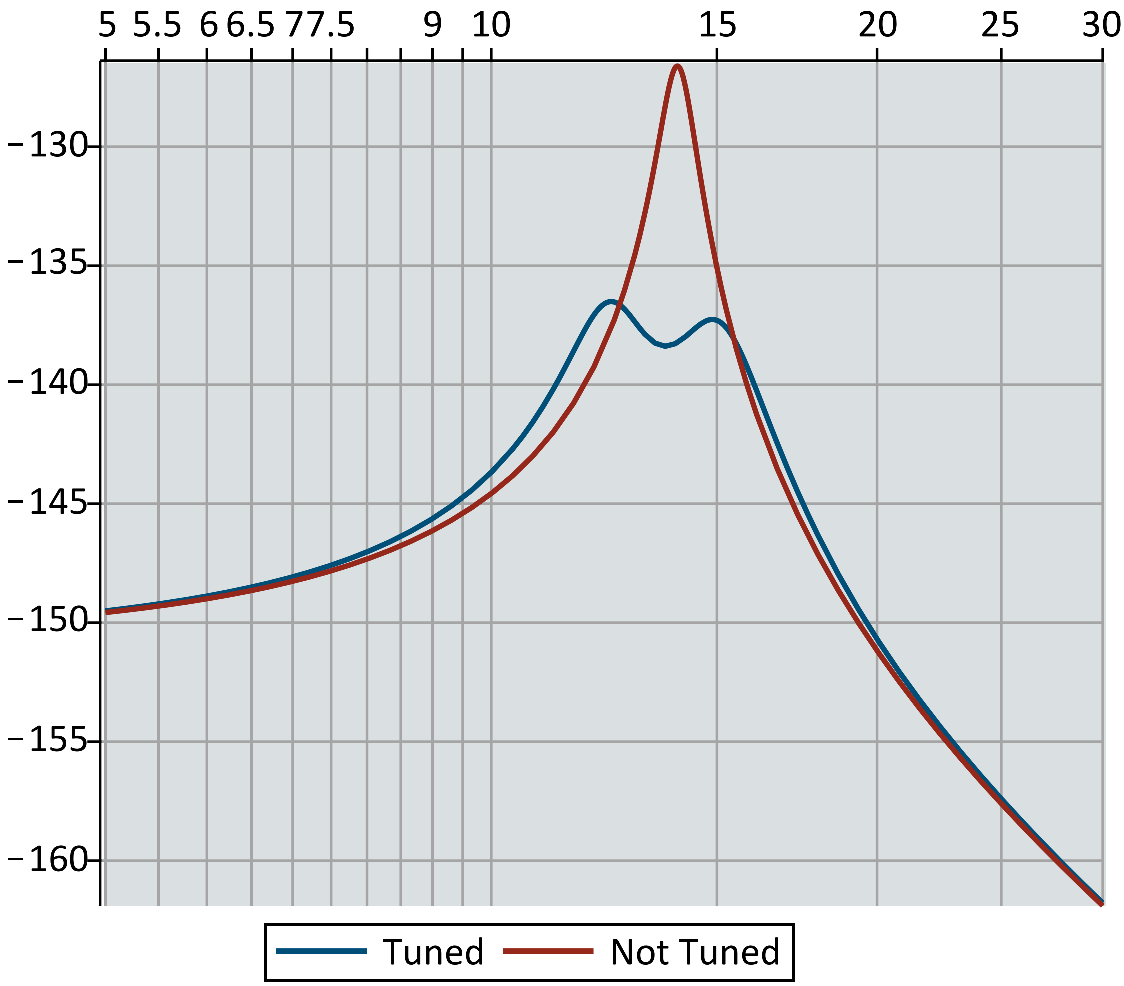
Response with tuned mass damper:

> $p1 := MagnitudePlot(sys, range = 5..30, parameters = params_{TMD}, color = Color("RGB", [0/255, 79/255, 121/255]), legend = "Tuned") :$

Response with no tuned mass damper:

> $p2 := MagnitudePlot(sys, range = 5..30, parameters = params_{noTMD}, color = Color("RGB", [150/255, 40/255, 27/255]), legend = "Not Tuned") :$

> plots:-*display*(p1, p2, size = [800, 400], thickness = 2, axesfont = [Calibri], labelfont = [Calibri], background = Color("RGB", [218/255, 223/255, 225/255]), legendstyle = [font = [Calibri]])



Dynamic Response

Assume that the system is perturbed at the natural frequency of the system.

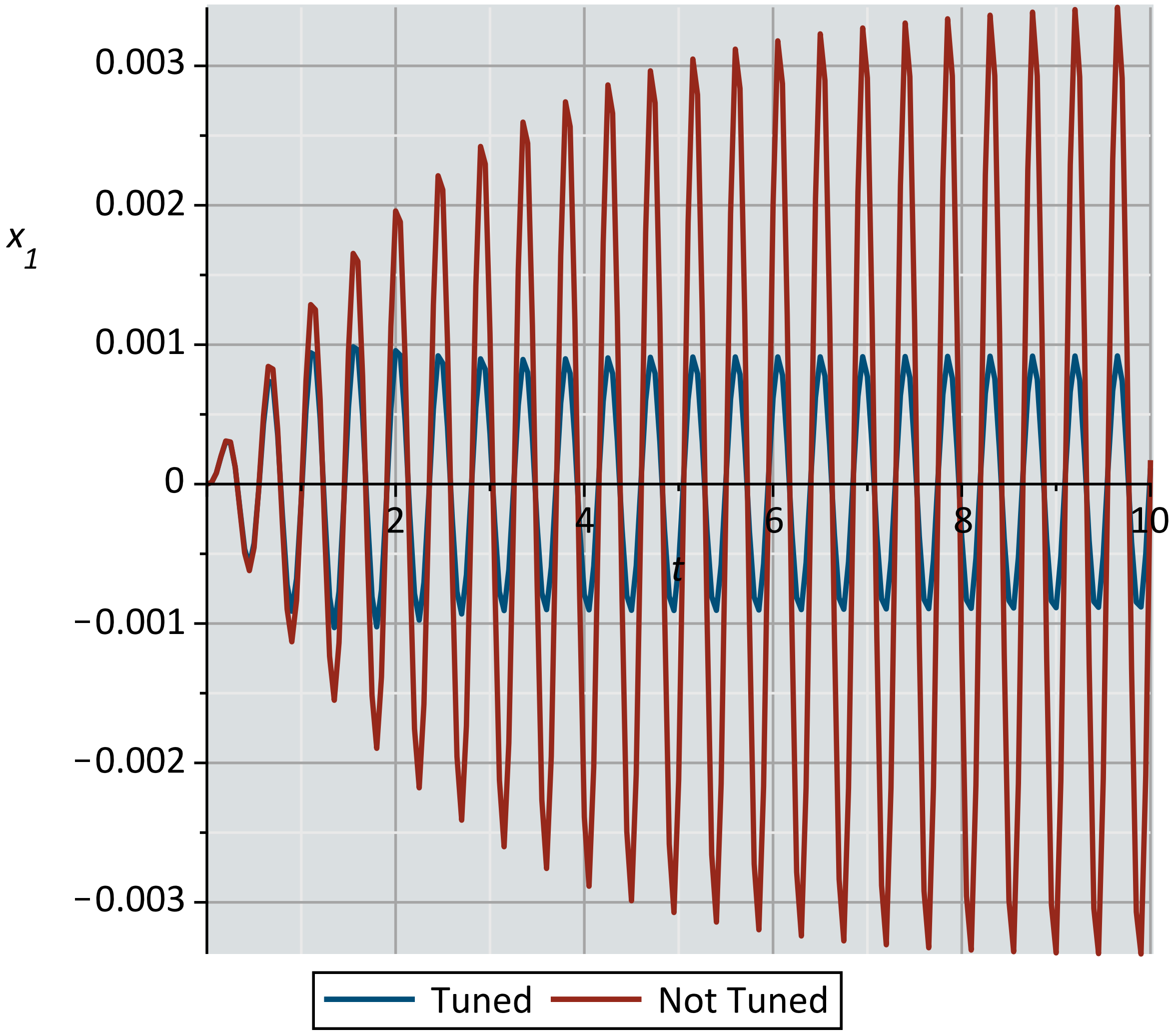
> $f_{nat} := \text{eval}\left(\omega_1, \left[params_{main}\right]\right)$

$$f_{nat} := 13.98492872 \tag{6.1}$$

> $p3 := \text{ResponsePlot}\left(sys, 7500 \sin\left(f_{nat} \cdot t\right), parameters = params_{TMD}, color = \text{Color}\left(\text{"RGB"}, [0/255., 79/255, 121/255]\right), legend = \text{"Tuned"}\right):$

> $p4 := \text{ResponsePlot}\left(sys, 7500 \sin\left(f_{nat}\, t\right), parameters = params_{noTMD}, color = \text{Color}\left(\text{"RGB"}, [150/255, 40/255, 27/255]\right), legend = \text{"Not Tuned"}\right):$

> plots:-*display*(p3, p4, axesfont = [Calibri], thickness = 2, size = [800, 400], gridlines, axesfont = [Calibri], labelfont = [Calibri], background = Color("RGB", [218/255, 223/255, 225/255]), legendstyle = [font = [Calibri]])



0.003
0.002
0.001
0
−0.001
−0.002
−0.003
x_1
2
4
6
8
10
t
Tuned
Not Tuned

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