Sunspot Periodicity

Introduction

This application will find the periodicity of sunspots with two separate approaches:
- Frequency domain analysis
- Autocorrelation

Both approaches should yield the same result.

```
> with(SignalProcessing):
  with(plots):
  with(DataSets):
```

International Sunspot Data

The following data set contains mean international sunspot numbers from the year 1700 to present.

The first column contains the "Year", while the second column represents the "Annual Mean Sunspot Number".

```
> DataReference := Reference("quandl", "SIDC/SUNSPOTS_A")
```

$$DataReference := \begin{bmatrix} \textit{Data set} \\ \textit{Total Sunspot Numbers (Annual)} \\ \textit{Quandl SIDC/SUNSPOTS\_A} \\ \textit{up to 319 rows (annual), 4 columns} \\ \textit{1700-12-31 - 2018-12-31} \end{bmatrix} \quad \textbf{(2.1)}$$

```
> data := convert(DataReference, Matrix):
```

	1	2
1	"1700-12-31"	8.3
2	"1701-12-31"	18.3
3	"1702-12-31"	26.7
4	"1703-12-31"	38.3
5	"1704-12-31"	60.0
6	"1705-12-31"	96.7
7	"1706-12-31"	48.3
8	"1707-12-31"	33.3

Data Set details:

> *DocumentTools*:-*Tabulate*(["Notes", *GetDescription*(*DataReference*)], *weights* = [20, 80])

"Tabulate" **(2.2)**

Notes	Since 1981, the Royal Observatory of Belgium harbours the Sunspot Index Data center, the World data center for the Sunspot Index. Recently, the Space Weather forecast center of Paris-Meudon was transferred and added to the activities of the SIDC. Moreover, a complete archive of all images of the SOHO instrument EIT has become available at the SIDC. SIDC-team, World Data Center for the Sunspot Index Royal Observatory of Belgium, Monthly Report on the International Sunspot Number online catalogue of the sunspot index: http://www.sidc.be/sunspot-data. Yearly mean total sunspot number obtained by taking a simple arithmetic mean of the daily total sunspot number over all days of each year. (NB: in early years in particular before 1749, the means are computed on only a fraction of the days in each year because on many days, no observation is available).</br>

Plot the Data

> *SunspotNumber* := *data*[.., 2] :

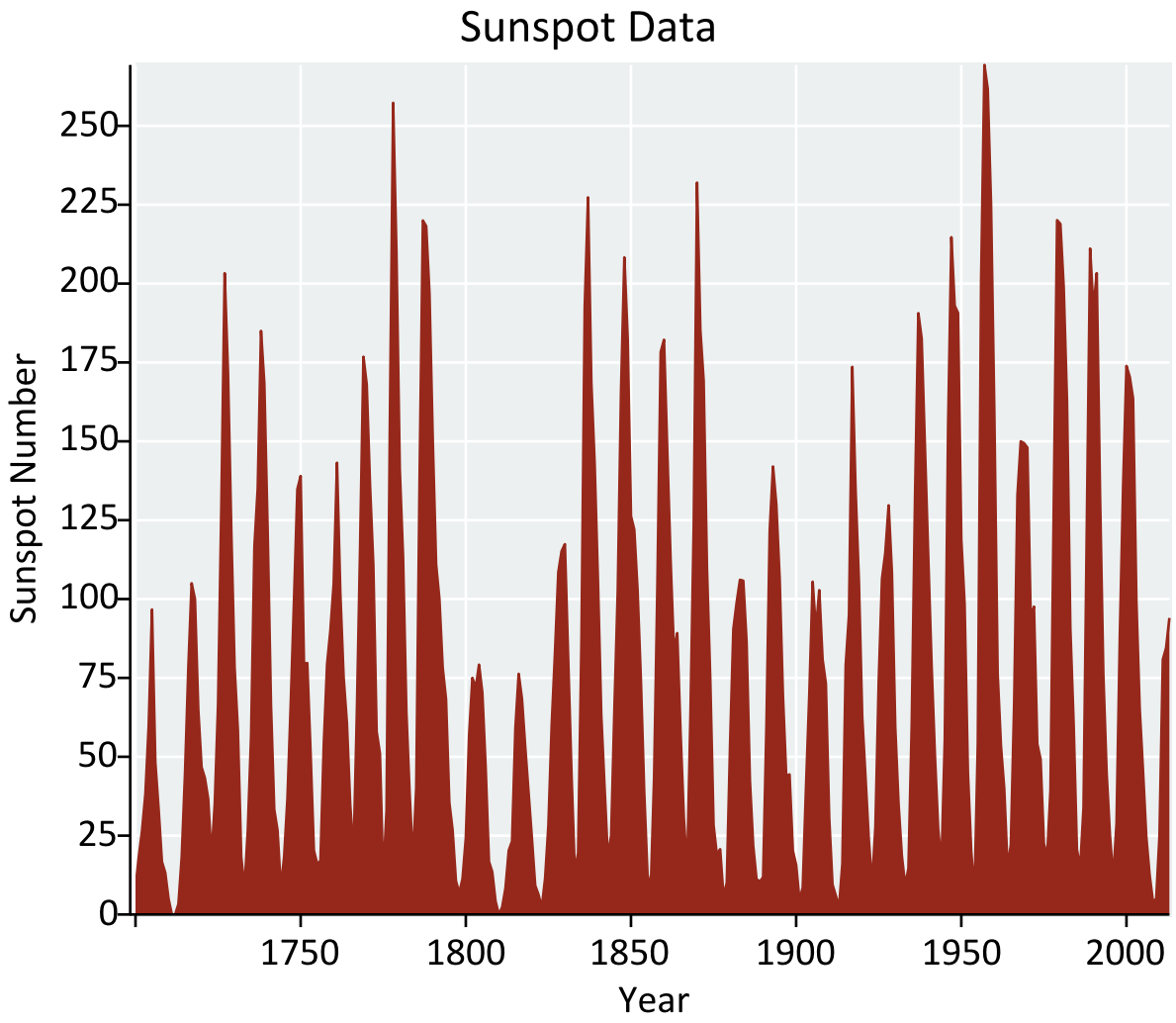
> *ParseYear* := *x* → *StringTools*:-*ParseTime*("%Y-%m-%d", *x*):-*year* :

> *Year* := *ParseYear*~(*data*[.., 1]) :

> *plot*([*seq*([$Year_i$, $SunspotNumber_i$], *i* = 1 ..314)], labels = ["Year", "Sunspot Number"],

labeldirections = [horizontal, vertical], title = "Sunspot Data", titlefont = [Calibri, 14],

thickness = 0, filled = true, size = [800, 400], *axesfont* = [*Calibri*], *labelfont* = [*Calibri*], *color* = $RGB\left(\frac{150}{255}, \frac{40}{255}, \frac{27}{255}\right)$, *transparency* = 0, *background* = *ColorTools*:-*Color*("RGB", $\left[\frac{236}{255}, \frac{240}{255}, \frac{241}{255}\right]$), *axis* = [*gridlines* = [10, *color* = *RGB*(1, 1, 1)]])



Periodicity via Fourier Transformation to the Frequency Domain

Now, calculate the period using a Fast Fourier Transform (FFT) of the first 2^8 data points:

> *fSunspots* := *FFT*(*SunspotNumber*[1..2^8]) :

Plot the power spectrum:

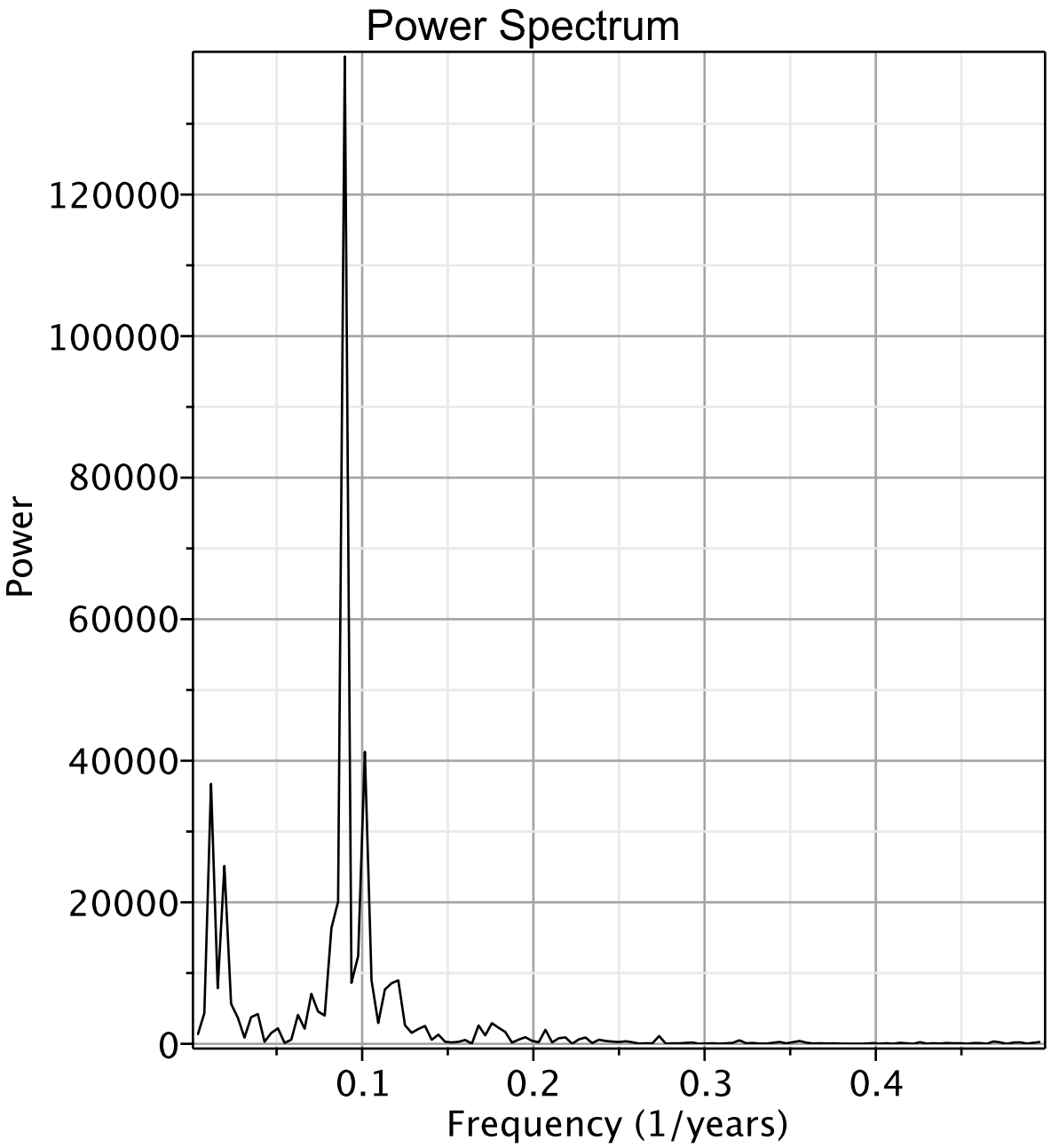
> *samplingRate* := 1 :

> *psSunspots* := *PowerSpectrum*(*fSunspots*) :

> *SignalPlot*(*psSunspots*[2..$\frac{numelems(psSunspots)}{2}$], *samplerate* = 2^8, *labels*

= ["Frequency (1/years)", "Power"], *title* = "Power Spectrum", *titlefont* = [*Arial*, 14], *size*

= [800, "golden"])



Note the peak at a frequency of 0.09 years^{-1} . Try zooming in and using the point probe to confirm the value of this peak frequency.

The period is the reciprocal of the peak frequency.

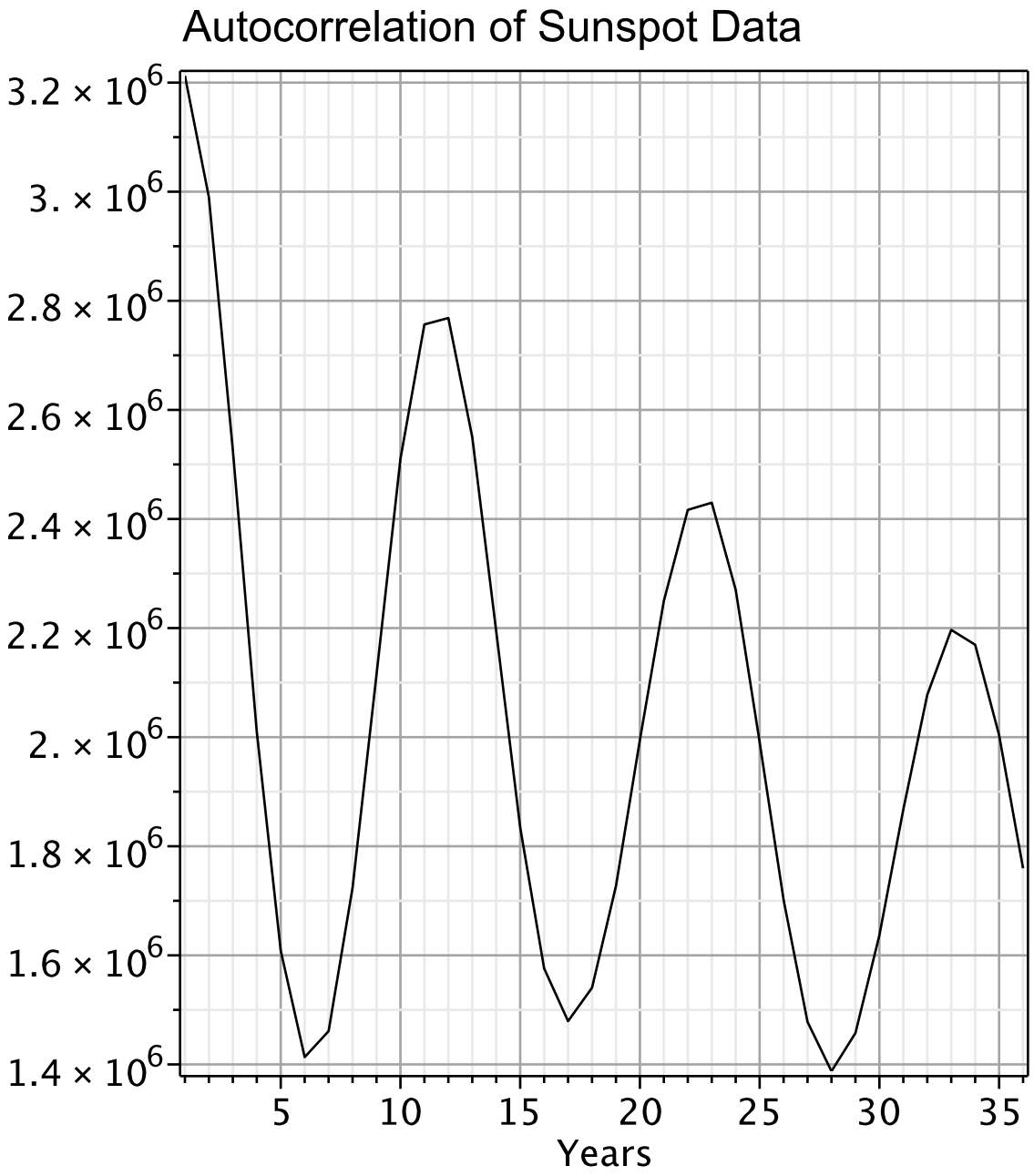
> $period := \frac{1}{0.09}$

$$period := 11.11111111 \quad \textbf{(4.1)}$$

Hence, the predicted periodicity is approximately 11 years.

Periodicity via Autocorrelation

```
> aSunspotNumber := AutoCorrelation( SunspotNumber ) :
> SignalPlot( aSunspotNumber[ 1 ..36 ], labels = [ "Years", "" ], title
      = "Autocorrelation of Sunspot Data", titlefont = [ Arial, 14 ], size = [ 800, "golden" ] );
```



Here the first peak is at 11 years, indicating that the periodicity of sunspots is approximately 11 years. This confirms the period predicted by the Fourier Transform approach.