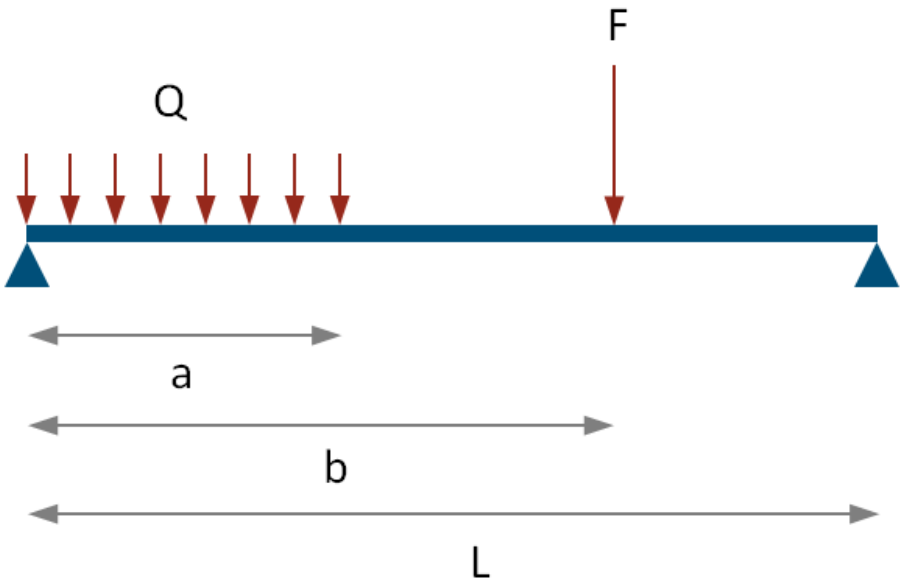
Deflection of a Beam with Distributed and Point Load

Introduction

This application will derive an explicit expression for the deflection of a beam with a distributed load and a point load.



Governing Equations

> *restart* :

The Euler-Bernoulli equation

> $de := EI \cdot \frac{d^4}{dx^4} w(x) = q(x)$:

Initial and boundary conditions

> $ibc := w(0) = 0, w(L) = 0, (D@@2)(w)(0) = 0, (D@@2)(w)(L) = 0$:

Distributed load and point load

> $q := x \to Q \cdot (1 - \text{Heaviside}(x - a)) + F \cdot \text{Dirac}(x - b)$:

Solution of the Differential Equation

Solve the differential equation together with the initial/boundary conditions and the load distribution to get an explicit expression for the beam deflection.

> $deSol := dsolve(\{de, ibc\}, w(x))$:
$deflection := simplify(rhs(deSol), symbolic)$

$$deflection := \frac{1}{144\,EI\,L}\left(-Q\,x\,(L-x)\,(L+x)\,(L-a)^4\,\text{Dirac}(1, L-a) + 4\,F\,x\,(L-x)\,(L \right. \quad \textbf{(3.1)}$$

$$+x)\,(L-b)^3\,\text{Dirac}(1, L-b) - 8\,Q\,x\,(L-x)\,(L+x)\,(L-a)^3\,\text{Dirac}(L-a)$$

$$-6\,Q\,x\,(L-a)^2\left(L^2+2\,L\,a-a^2-2\,x^2\right)\text{Heaviside}(L-a) + 24\,F\,x\,(L-x)\,(L$$

$$+x)\,(L-b)^2\,\text{Dirac}(L-b) + 48\,x\,F\left(-\frac{x^2}{2} + b\left(L-\frac{b}{2}\right)\right)(L-b)\,\text{Heaviside}(L-b)$$

$$-6\,L\,Q\,(-x+a)^4\,\text{Heaviside}(x-a) - 24\,F\,L\,(-x+b)^3\,\text{Heaviside}(x-b)$$

$$-48\left(\frac{a^2\,Q\left(L\,x-\frac{1}{4}\,a^2-\frac{1}{2}\,x^2\right)\text{Heaviside}(-a)}{2} + F\,b\left(L\,x-\frac{1}{2}\,b^2\right.\right.$$

$$\left.\left.-\frac{1}{2}\,x^2\right)\text{Heaviside}(-b) - \frac{Q\,x\,L\left(L^2+L\,x-x^2\right)}{8}\right)(L-x)\Bigg)$$

Derive the moment and shear distribution.

> $moment := EI \cdot diff(deflection, x, x)$

$$moment := \frac{1}{144\,L}\left(2\,Q\,x\,(L-a)^4\,\text{Dirac}(1, L-a) - 2\,Q\,(L-x)\,(L-a)^4\,\text{Dirac}(1, L-a)\right. \quad \textbf{(3.2)}$$

$$-144\,F\,L\,(-x+b)\,\text{Heaviside}(x-b) - 24\,F\,L\,(-x+b)^3\,\text{Dirac}(1, x-b) + 48\,a^2\,Q\,(L$$

$$-x)\,\text{Heaviside}(-a) + 96\,F\,b\,(L-x)\,\text{Heaviside}(-b) - 12\,Q\,L\left(L^2+L\,x-x^2\right)$$

$$-12\,Q\,x\,L\,(L-2\,x) + 2\,Q\,(L+x)\,(L-a)^4\,\text{Dirac}(1, L-a) - 8\,F\,(L+x)\,(L$$

$$-b)^3\,\mathrm{Dirac}(1,L-b)+16\,Q\,(L+x)\,(L-a)^3\,\mathrm{Dirac}(L-a)+72\,Q\,(L$$

$$-a)^2\,x\,\mathrm{Heaviside}(L-a)-48\,F\,(L+x)\,(L-b)^2\,\mathrm{Dirac}(L-b)-144\,F\,x\,(L$$

$$-b)\,\mathrm{Heaviside}(L-b)+48\,L\,Q\,(-x+a)^3\,\mathrm{Dirac}(x-a)-72\,L\,Q\,(-x$$

$$+a)^2\,\mathrm{Heaviside}(x-a)-6\,L\,Q\,(-x+a)^4\,\mathrm{Dirac}(1,x-a)+144\,F\,L\,(-x$$

$$+b)^2\,\mathrm{Dirac}(x-b)-48\left(-\frac{Q\,a^2\,\mathrm{Heaviside}(-a)}{2}-F\,b\,\mathrm{Heaviside}(-b)\right.$$

$$\left.-\frac{Q\,L\,(L-2\,x)}{4}+\frac{Q\,x\,L}{4}\right)(L-x)+8\,F\,(L-x)\,(L-b)^3\,\mathrm{Dirac}(1,L-b)$$

$$-8\,F\,x\,(L-b)^3\,\mathrm{Dirac}(1,L-b)-16\,Q\,(L-x)\,(L-a)^3\,\mathrm{Dirac}(L-a)$$

$$+16\,Q\,x\,(L-a)^3\mathrm{Dirac}(L-a)+48\,F\,(L-x)\,(L-b)^2\,\mathrm{Dirac}(L-b)-48\,F\,x\,(L$$
$$-b)^2\,\mathrm{Dirac}(L-b)\Big)$$

> $shear := diff(moment, x)$

$$shear := \frac{1}{144\,L}\Big(-144\,F\,b\,\mathrm{Heaviside}(-b)-72\,Q\,a^2\,\mathrm{Heaviside}(-a)+6\,Q\,(L \tag{3.3}$$
$$-a)^4\,\mathrm{Dirac}(1,L-a)+144\,F\,L\,\mathrm{Heaviside}(x-b)-36\,Q\,L\,(L-x)-36\,Q\,L\,(L$$
$$-2\,x)-24\,F\,(L-b)^3\,\mathrm{Dirac}(1,L-b)+48\,Q\,(L-a)^3\,\mathrm{Dirac}(L-a)+72\,Q\,(L$$
$$-a)^2\,\mathrm{Heaviside}(L-a)-144\,F\,(L-b)^2\,\mathrm{Dirac}(L-b)-144\,F\,(L-b)\,\mathrm{Heaviside}(L$$
$$-b)+72\,L\,Q\,(-x+a)^3\,\mathrm{Dirac}(1,x-a)-216\,L\,Q\,(-x+a)^2\,\mathrm{Dirac}(x-a)$$
$$+144\,L\,Q\,(-x+a)\,\mathrm{Heaviside}(x-a)-6\,L\,Q\,(-x+a)^4\,\mathrm{Dirac}(2,x-a)-432\,F\,L\,($$
$$-x+b)\,\mathrm{Dirac}(x-b)-24\,F\,L\,(-x+b)^3\,\mathrm{Dirac}(2,x-b)+216\,F\,L\,(-x$$
$$+b)^2\,\mathrm{Dirac}(1,x-b)+36\,Q\,x\,L\Big)$$

Plot the Deflection, Moment, and Shear

Assign parameters.

> $Q := -12:$
$F := -10:$
$L := 20:$
$a := 3:$
$b := 5:$
$EI := 10000:$

Plot deflection, moment, and shear.

```
> plot(deflection, x = 0 ..L, size = [1000, 400], axesfont = [Calibri], title = "Deflection", labels
    = ["Distance along beam", "Deflection"], labeldirections = [horizontal, vertical], labelfont
    = [Calibri], titlefont = [Calibri, 16, bold], background = ColorTools:-Color("RGB", [218
    /255, 223/255, 225/255]), axis = [gridlines = [color = ColorTools:-Color("RGB", [1, 1,
    1])]])
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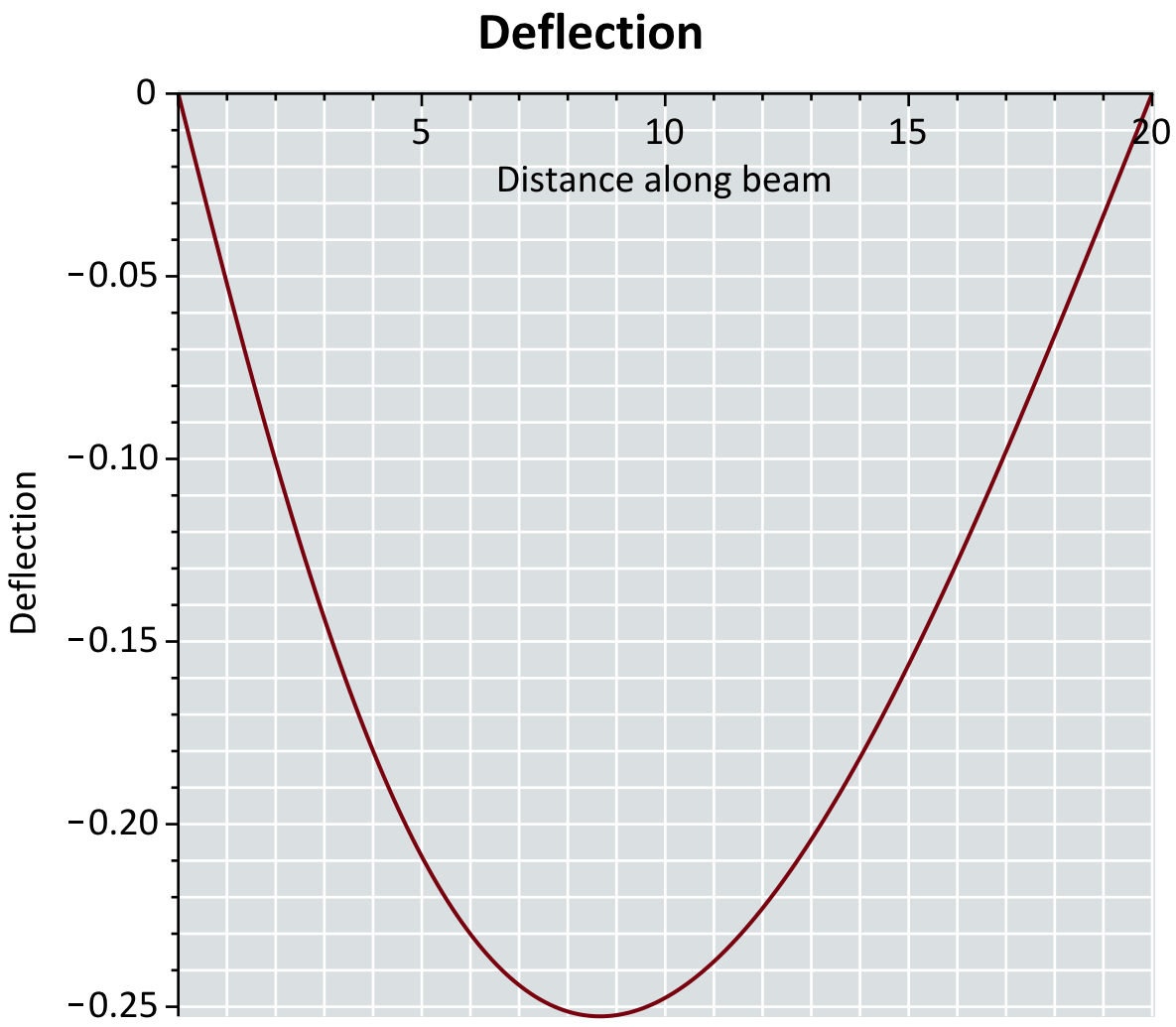


```
> plot(moment, x = 0 ..L, size = [1000, 400], axesfont = [Calibri], title = "Moment", labels
    = ["Distance along beam", "Moment"], labeldirections = [horizontal, vertical], labelfont
    = [Calibri], titlefont = [Calibri, 16, bold], background = ColorTools:-Color("RGB", [218
    /255, 223/255, 225/255]), axis = [gridlines = [color = ColorTools:-Color("RGB", [1, 1,
    1])]])
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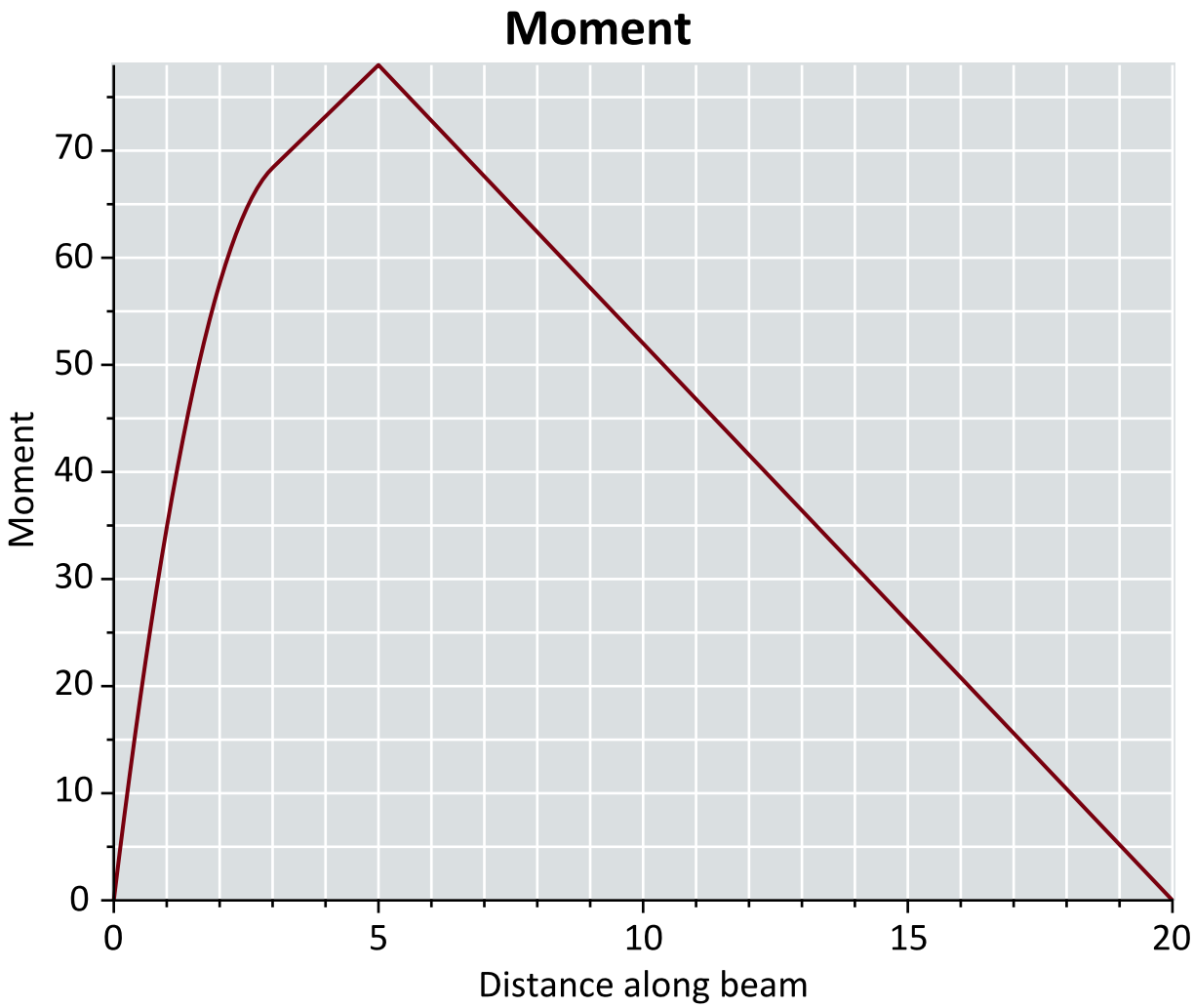

Shear Distribution

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> plot(shear, x = 0 ..L, size = [1000, 400], axesfont = [Calibri], title = "Shear", labels
  = ["Distance along beam", "Shear"], labeldirections = [horizontal, vertical], labelfont
  = [Calibri], titlefont = [Calibri, 16, bold], background = ColorTools:-Color("RGB", [218
  /255, 223/255, 225/255]), axis = [gridlines = [color = ColorTools:-Color("RGB", [1, 1,
  1])]])
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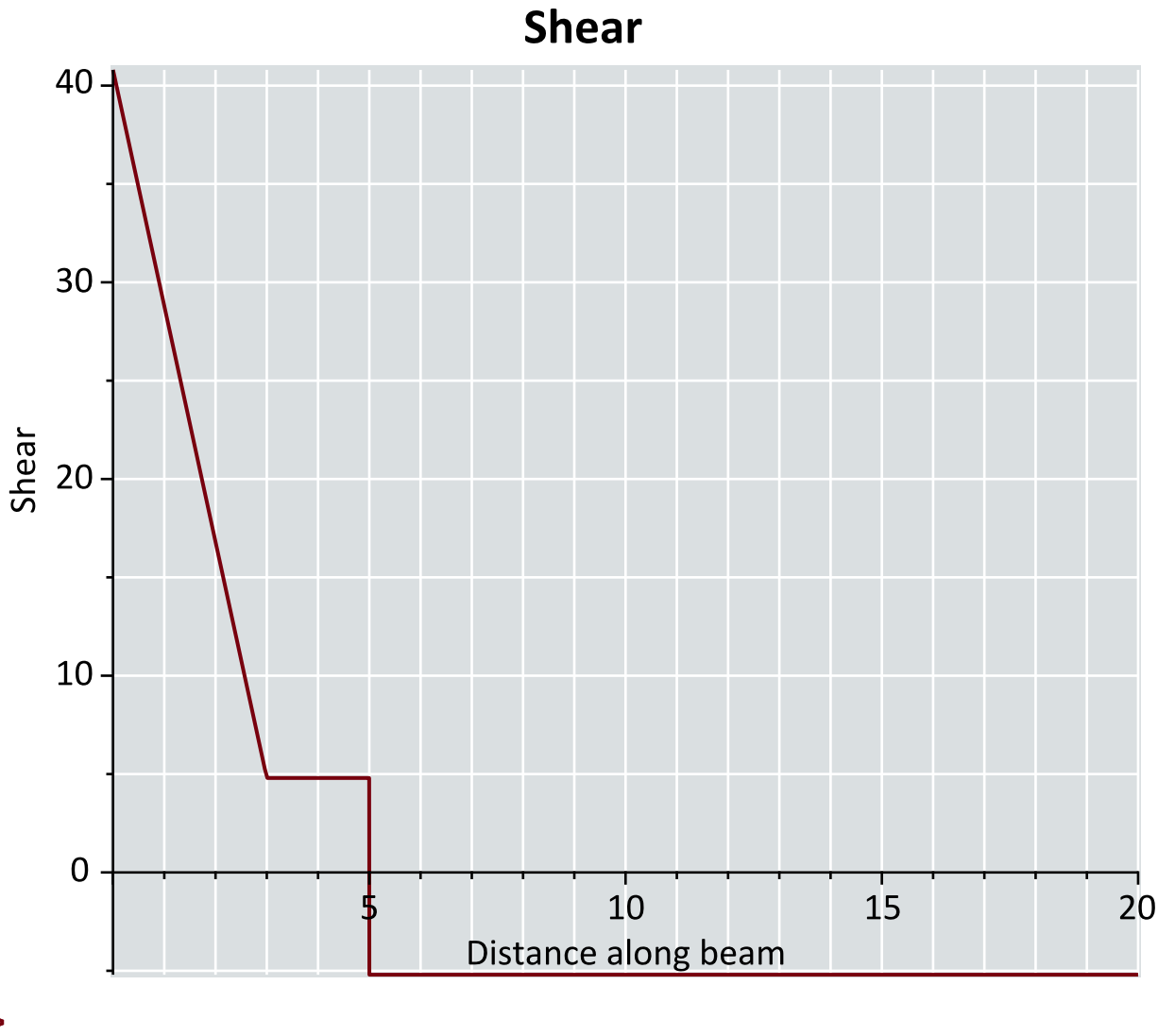

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