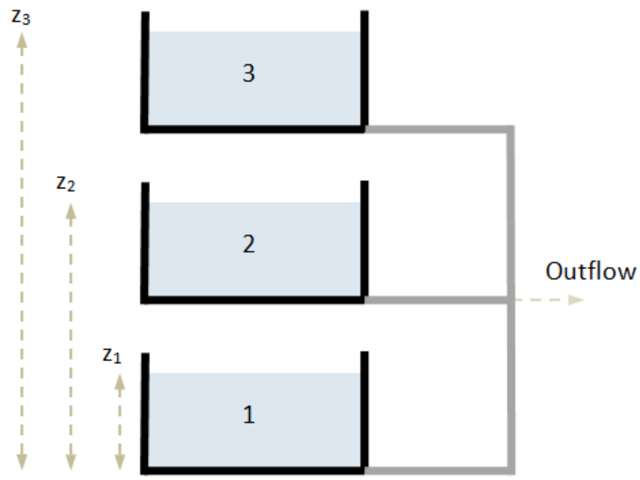
Three Reservoir Problem

Introduction

Three reservoirs at different elevations are connected through a piping network at a single point, with an outflow from the common junction. This application will calculate the flow rates, flow directions, and head at the common junction.



The Bernoulli equation (ignoring the losses associated with pipe fittings) for Reservoir 1 is

$$z_1 = f_1 \frac{L_1}{D_1} \frac{V_1^2}{2\,g} + H$$

where z_1 is the reservoir elevation, f_1 is the friction factor, L_1 is the pipe length, D_1 is the pipe diameter, V_1 is the liquid velocity, and H is the head at the common junction. However, since flow can be either into or out of a reservoir (that is, a positive or negative velocity), we rewrite the Bernoulli equation thus

$$z_1 = f \frac{L_1}{D_1} \frac{V_1 |V_1|}{2\,g} + H$$

Similar equations can be defined for the other reservoirs. These equations, along with the continuity equation,

$$Q_1 + Q_2 + Q_3 = Q_{outflow}$$

and a correlation for the friction factor, can be used to find the flow rates, flow directions, and the head at the junction.

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> restart :
  with(ThermophysicalData) :
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Parameters

Liquid viscosity and density

> $\mu := \text{Property}(\text{"viscosity"}, \text{"water"}, \text{"pressure"} = 1\,\text{atm}, \text{"temperature"} = 298\,\text{K})$
> $\rho := \text{Property}(\text{"density"}, \text{"water"}, \text{"pressure"} = 1\,\text{atm}, \text{"temperature"} = 298\,\text{K})$

$$893.07 \times 10^{-6}\,\text{Pa}\,\text{s}$$

$$997.09\,\frac{\text{kg}}{\text{m}^3} \qquad \textbf{(2.1)}$$

Reservoir elevations

> $z_1 := 90\,\text{m}$:
> $z_2 := 85\,\text{m}$:
> $z_3 := 60\,\text{m}$:

Lengths, diameters, and cross-sectional areas of the pipes connecting the reservoirs to the junction

> $L_1 := 2000\,\text{m}$:
> $L_2 := 1500\,\text{m}$:
> $L_3 := 3000\,\text{m}$:

> $D_1 := 0.3\,\text{m}$:
> $D_2 := 0.25\,\text{m}$:
> $D_3 := 0.25\,\text{m}$:

> $A_1 := \pi D_1^2/4$:
> $A_2 := \pi D_2^2/4$:
> $A_3 := \pi D_3^2/4$:

Roughness of the pipe

> $e := 0.0005\,\text{m}$:

Outflow from the junction

> $Q_{outflow} := 0.03\,\text{m}^3\,\text{s}^{-1}$:

Gravitational constant

> $g := 9.81\,\text{m}\,\text{s}^{-2}$:

Bernoulli Equations for the Three Reservoir System

Bernoulli analysis of the system

> sys :=
$z_1 = fric_1 L_1 Q_1 |Q_1| / (2\,g\,D_1 A_1^2) + H,$
$z_2 = fric_2 L_2 Q_2 |Q_2| / (2\,g\,D_2 A_2^2) + H,$

$z_3 = fric_3 L_3 Q_3 |Q_3| / (2\, g\, D_3 A_3^2) + H,$

$Q_1 + Q_2 + Q_3 = Q_{outflow},$

$Rey_1 = 4\, abs(Q_1)\, \rho / (\pi D_1 \mu),$
$Rey_2 = 4\, abs(Q_2)\, \rho / (\pi D_2 \mu),$
$Rey_3 = 4\, abs(Q_3)\, \rho / (\pi D_3 \mu),$

$fric_1 = piecewise\left(Rey_1 < 2500, 64 / Rey_1, 1 / \left(1.8\, \log10\left((e/(3.7\, D_1))^{1.11} + 6.9 / Rey_1\right)\right)^2\right),$
$fric_2 = piecewise\left(Rey_2 < 2500, 64 / Rey_2, 1 / \left(1.8\, \log10\left((e/(3.7\, D_2))^{1.11} + 6.9 / Rey_2\right)\right)^2\right),$
$fric_3 = piecewise\left(Rey_3 < 2500, 64 / Rey_3, 1 / \left(1.8\, \log10\left((e/(3.7\, D_3))^{1.11} + 6.9 / Rey_3\right)\right)^2\right):$

> $estimates := H = 1\ m, Q_1 = 0.1\ m^3 s^{-1}, Q_2 = 0.1\ m^3 s^{-1}, Q_3 = 0.1\ m^3 s^{-1}, Rey_1 = 1, Rey_2 = 1, Rey_3 = 1, fric_1 = 1, fric_2 = 1, fric_3 = 1 :$

Calculation of Flow Rates, Flow Directions, and Junction Head

Positive flow rates indicate flow out of a reservoir, while negative flow rates indicate flow into a reservoir

> $Digits := 20 :$

> $results := fsolve(\{sys\}, \{estimates\})$

$$\left\{H = 8.31 \times 10^{1}\ m, Q_1 = 6.67 \times 10^{-2}\ \frac{m^3}{s}, Q_2 = 2.49 \times 10^{-2}\ \frac{m^3}{s}, Q_3 = -6.16 \times 10^{-2}\ \frac{m^3}{s}, Rey_1 = 3.16 \times 10^{5}, Rey_2 = 1.42 \times 10^{5}, Rey_3 = 3.50 \times 10^{5}, fric_1 = 2.29 \times 10^{-2}, fric_2 = 2.46 \times 10^{-2}, fric_3 = 2.39 \times 10^{-2}\right\} \tag{4.1}$$