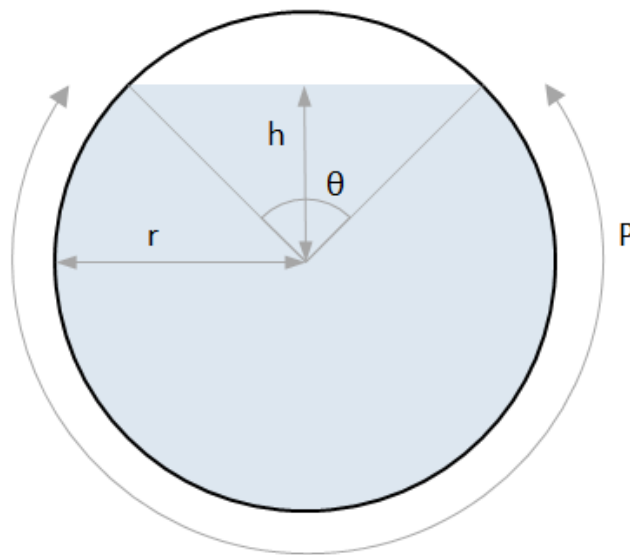


Maximum Flow Rate in Open-Channel Flow for a Circular Pipe

▼ Introduction

This application determines the greatest attainable water flow rate in a partially filled circular pipe.



It uses the [Manning formula](#) to determine the flow rate in the open-channel flow of water:

$$Q = \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}},$$

where:

- Q is the flow rate
- n is an empirical coefficient
- A is the cross-sectional area of flow
- R is the hydraulic radius
- S_0 is the incline of the channel

An equation that represents the hydraulic radius of a partially filled circular pipe is derived and substituted into the Manning formula. The resulting equation is then optimized to find the maximum flow rate.

▼ Manning Formula for a Circular Pipe

> restart :

Manning formula:

$$> Q := \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}} :$$

For a partially filled circular pipe, the flow area (the blue shaded area in the diagram) is:

$$> A := \pi \cdot r^2 - r^2 \cdot \frac{(\theta - \sin(\theta))}{2} :$$

The wetted perimeter is given by the following formula.

$$> P := 2 \cdot \pi \cdot r - r \cdot \theta :$$

Hence, the hydraulic radius, R , is:

$$> R := \frac{A}{P}$$

$$R := \frac{\pi r^2 - r^2 \left(\frac{\theta}{2} - \frac{\sin(\theta)}{2} \right)}{2 \pi r - r \theta} \quad (2.1)$$

The Manning formula then becomes:

> Q

$$\frac{1.49 \left(\pi r^2 - r^2 \left(\frac{\theta}{2} - \frac{\sin(\theta)}{2} \right) \right) \left(\frac{\pi r^2 - r^2 \left(\frac{\theta}{2} - \frac{\sin(\theta)}{2} \right)}{2 \pi r - r \theta} \right)^{2/3} \sqrt{S_0}}{n} \quad (2.2)$$

▼ Maximum Flow Rate for a Circular Pipe

Pipe radius:

$$> r := 3 :$$

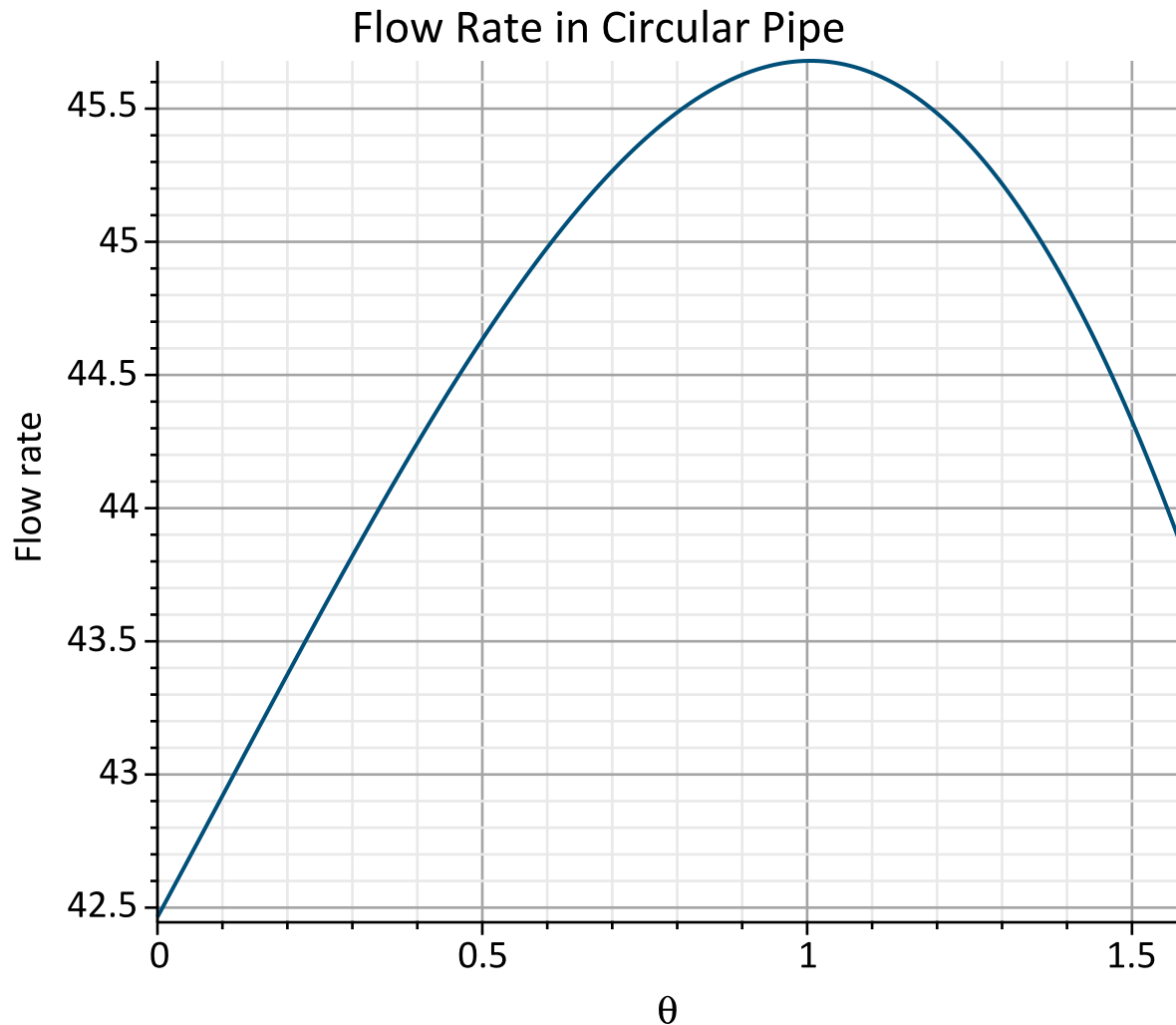
Incline of the channel:

$$> S_0 := 0.0001 :$$

Roughness coefficient:

$$> n := 0.013 :$$

```
> plot(Q,  $\theta = 0 \dots 0.5 \pi$ , labels = [" $\theta$ ", "Flow rate"], labeldirections = [horizontal, vertical], labelfont = [Calibri], title = "Flow Rate in Circular Pipe", titlefont = [Calibri, 14], size = [600, 400], axesfont = [Calibri], gridlines, color = ColorTools:-Color("RGB", [0/255, 79/255, 121/255]))
```



```
> res := Optimization:-Maximize(Q,  $\theta = 0 \dots 0.5 \pi$ )
res := [45.6796864427174, [ $\theta = 1.00507814259573$ ]] (1)
```

The maximum flow rate is...

```
>  $Q_{maxflow} := res[1]$ 
 $Q_{maxflow} := 45.6796864427174$  (2)
```

...when θ (in radians) is ...

```
>  $\theta_{maxflow} := rhs(res[2, 1]);$ 
 $\theta_{maxflow} := 1.00507814259573$  (3)
```

... and the flow depth is:

$$> h := r \cdot \cos\left(0.5 \theta_{maxflow}\right) : \\ h + r$$

5.62908731621774

(4)

>