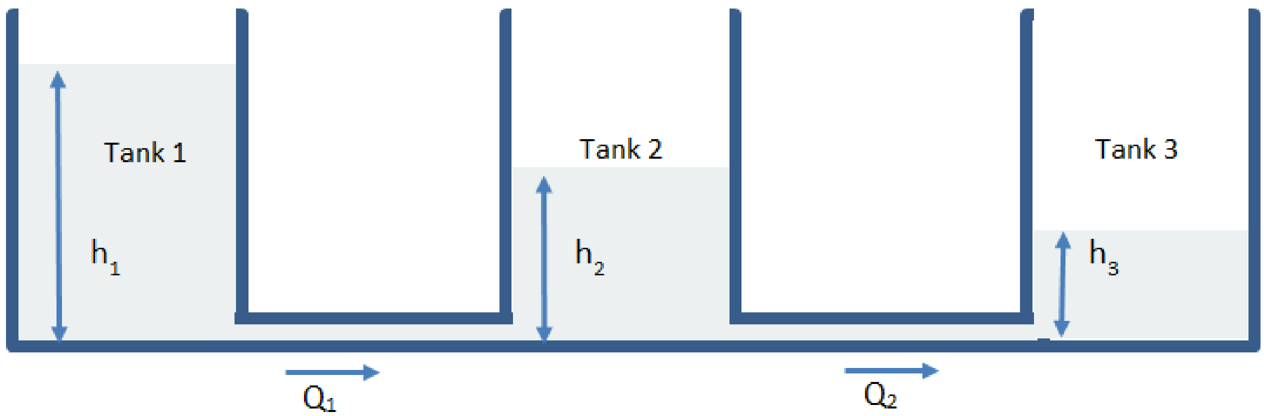
Interacting Tanks

Introduction

This worksheet models liquid flow between three tanks connected by two pipes (the first pipe connecting Tank 1 and 2, and the second pipe connecting Tank 2 and 3).



The flow is opposed by pipe friction, and the level of liquid in each tank oscillates to an equilibrium. Differential equations that describe the dynamic change in liquid height in each tank and a momentum balance are solved numerically.

> *restart*;

Physical Parameters

The cross-sectional area of the tanks:

> $A_1 := 1 : A_2 := 5 : A_3 := 2 :$

Diameter, length, and roughness of the pipe:

> $Dia := 0.3 : L := 100 : e := 0.001 :$

Density and viscosity of the liquid:

> $\rho := 1000 : \mu := 0.001 :$

Gravitational constant:

> $g := 9.81 :$

Momentum Balance

Friction Factor

```
> friction :=proc(Q)
local Rey, fL, fT :
  if type(Q, numeric) then
    Rey := 4 Q ρ / (evalf(π) Dia μ) :
    fL := 64 / Rey :
    fT := 1 / (1.8 log10(6.9/Rey + (e/(3.7 Dia))^1.11))^2 :

    if Rey > 0 and Rey < 2000 then
       return fL :
    elif Rey ≥ 2000 and Rey < 4000 then
      return fL + (fT − fL)·(Rey − 2000) / (4000 − 2000)
    elif Rey ≥ 4000 then
      return fT
    else
       return 0
    end if;
  else
    return 'friction'(Q)
  end if
end proc:
```

$$\mathrm{Rey} := \frac{4\,Q\,\rho}{\mathrm{evalf}(\pi)\ \mathrm{Dia}\,\mu}$$

$$\mathrm{fL} := \frac{64}{\mathrm{Rey}}$$

$$\mathrm{fT} := \frac{1}{\left(1.8\,\mathrm{log10}\left(\frac{6.9}{\mathrm{Rey}} + \left(\frac{e}{3.7\,\mathrm{Dia}}\right)^{1.11}\right)\right)^{2}}$$

$$\text{return } \mathrm{fL} + \frac{(\mathrm{fT} - \mathrm{fL})\cdot(\mathrm{Rey} - 2000)}{4000 - 2000}$$

Differential Equations

The rate of change of liquid height in Tank 1:

$$> height_1 := \frac{d}{dt} H_1(t) = -\frac{Q_1(t)}{A_1} :$$

The rate of change of liquid height in Tank 2:

$$> height_2 := \frac{d}{dt} H_2(t) = \frac{Q_1(t) - Q_2(t)}{A_2} :$$

The rate of change of liquid height in Tank 3:

$$> height_3 := \frac{d}{dt} H_3(t) = \frac{Q_2(t)}{A_3} :$$

A momentum balance:

> $momentumBalance_1 := \frac{d}{dt} Q_1(t) = \frac{\pi Dia^2 g H_1(t)}{4L} - \frac{\pi Dia^2 g H_2(t)}{4L} - \frac{2 \cdot friction(\text{abs}(Q_1(t)))\, \text{abs}(Q_1(t)) \cdot Q_1(t)}{\pi Dia^3}$:

> $momentumBalance_2 := \frac{d}{dt} Q_2(t) = \frac{\pi Dia^2 g H_2(t)}{4L} - \frac{\pi Dia^2 g H_3(t)}{4L} - \frac{2 \cdot friction(\text{abs}(Q_2(t)))\, \text{abs}(Q_2(t)) \cdot Q_2(t)}{\pi Dia^3}$:

The initial conditions:

> $initialConditions := Q_1(0) = 0, Q_2(0) = 0, H_1(0) = 1.5, H_2(0) = 1.2, H_3(0) = 2$:

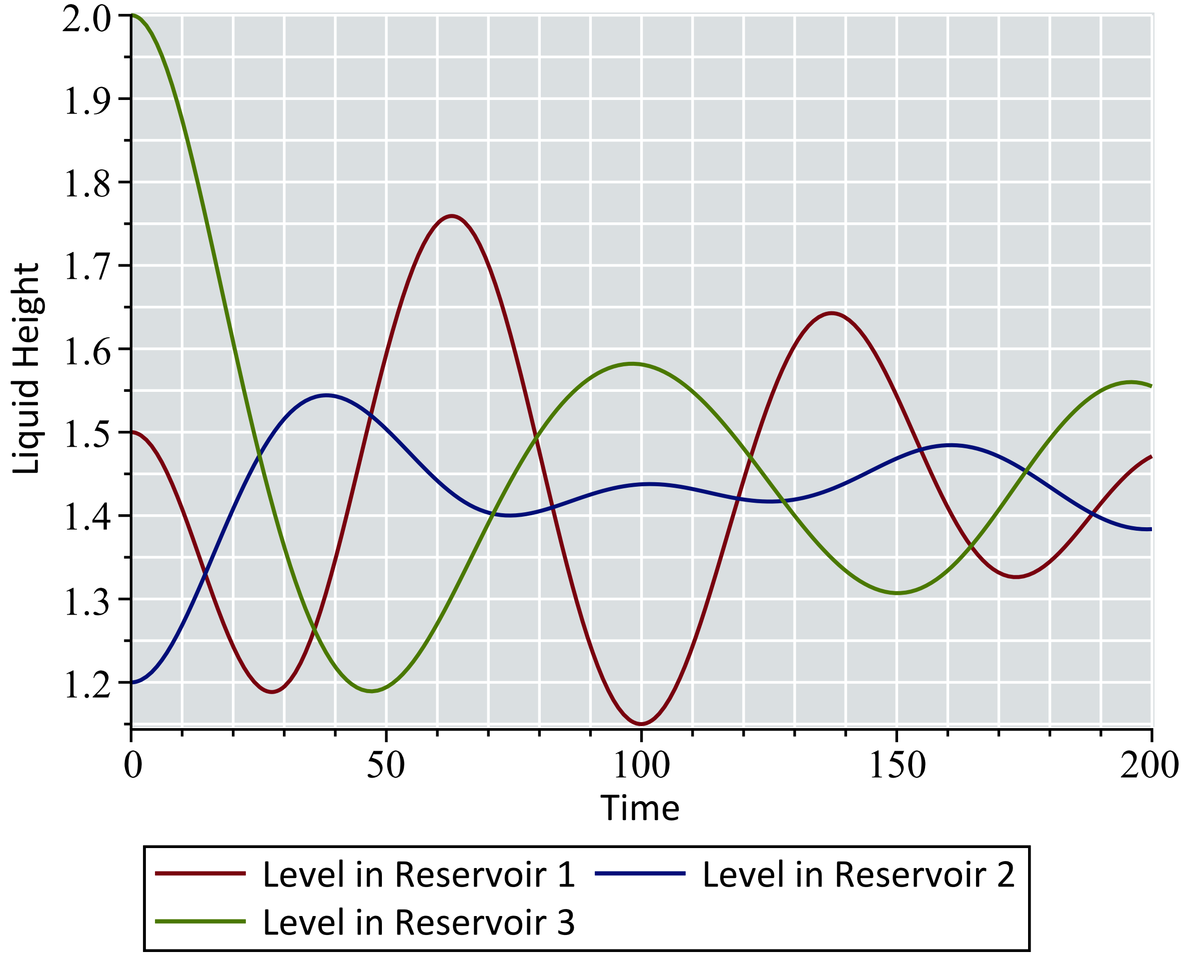
Numerical Solution of Governing Equations

> $res := dsolve(\{height_1, height_2, height_3, momentumBalance_1, momentumBalance_2, initialConditions\}, \{H_1(t), H_2(t), H_3(t), Q_1(t), Q_2(t)\}, numeric, output = listprocedure, known = friction)$:

> $H_1 := subs(res, H_1(t))$: $H_2 := subs(res, H_2(t))$: $H_3 := subs(res, H_3(t))$:
$Q_1 := subs(res, Q_1(t))$: $Q_2 := subs(res, Q_2(t))$:

Results

```
> plot([H1(t), H2(t), H3(t)], t = 0..200,
      legend = (["Level in Reservoir 1","Level in Reservoir 2", "Level in Reservoir 3"]),
      labels = (["Time","Liquid Height"]),
      labeldirections = ([horizontal, vertical]),
      labelfont = [Calibri],
      titlefont = [Calibri, 16, bold],
      background = ColorTools:-Color("RGB", [218/255, 223/255, 225/255]),
      legendstyle = [font = [Calibri]],
      axis = [gridlines = [color = ColorTools:-Color("RGB", [1, 1, 1])]],
      size = [1000, 400]);
```



Liquid Height
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
0
50
100
150
200
Time
Level in Reservoir 1
Level in Reservoir 2
Level in Reservoir 3