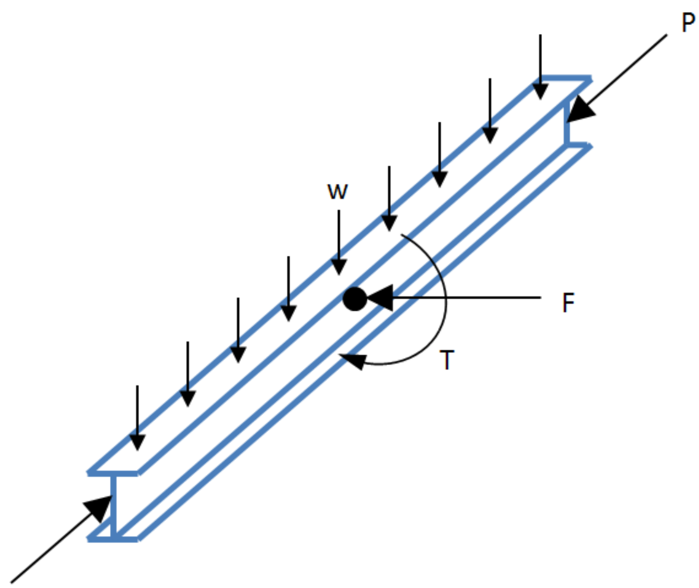
Torsional and Lateral Loading on W10X54 Beam

This application performs a design analysis on a simply supported beam with torsional loading for a W10X54 steel beam (as defined by the AISC Steel Shapes Database).



References
- Simplified Design for Torsional Loading of Rolled Steel Members, Lin, P.H., Engineering Journal, AISC, 1977
- 2010 Specification for Structural Steel Buildings (ANSI/AISC 360/10), Fourth Printing (https://www.aisc.org/content.aspx?id=2884)

with(AISCShapes) :
with(Units[Simple]) :

Steel Section Properties

Steel shape
shape := "W10X54" :

Warping constant
Cw := Property(shape, "Cw") $= 2320.0\ \mathrm{in}^6$

Torsional moment of inertia
JT := Property(shape, "J") $= 1.82\ \mathrm{in}^4$

Elastic section modulus about the x-axis
Sx := Property(shape, "Sx") $= 60.0\ \mathrm{in}^3$

Elastic section modulus about the y-axis
Sy := Property(shape, "Sy") $= 20.6\ \mathrm{in}^3$

Cross sectional area of member
A := Property(shape, "A") $= 15.8\ \mathrm{in}^2$

Plastic section modulus about the x-axis
Zx := Property(shape, "Zx") $= 66.6\ \mathrm{in}^3$

Moment of inertia about the x-axis

$Iy := Property(shape, "Iy") \quad = 103.0\ in^4$

Overall depth of member

$d := Property(shape, "d") \quad = 10.1\ in$

Radius of yration about the x-axis = srqt(Ix/A)

$rx := Property(shape, "rx") \quad = 4.37\ in$

Moment of inertia about the x-axis

$Ix := Property(shape, "Ix") \quad = 303.0\ in^4$

Parameters

Gravity distributed load

$w := 1.15\ kipf\ ft^{-1}$:

Lateral point load at the middle

$F := 5\ kipf$:

Beam length

$L := 15 ft$:

Beam yield stress

$Fy := 50\ ksi$:

Axial horizontal unbraced length

$Ly := 7.5\ ft$:

Young's modulus

$E := 29000\ ksi$:

Torsion at mid-span

$T := 5.1\ ft\ kipf$:

Axial Load

$P := 96\ kipf$:

Vertical bending unbraced length

$Lb := 15\ ft$:

Axial vertical unbraced length

$Lx := 15\ ft$:

Shear modulus

$G := 11200\ ksi$:

Torsional property (Phillip, 1977)

$$\lambda := \sqrt{\frac{G \cdot JT}{E \cdot Cw}} = 0.69\ \frac{1}{m}$$

Governing Moments at Middle of Span

Flexural moments

$Mx := w L^2 / 8 \quad = 32.34\ foot\ kipf$

$My := F L / 4.0 \quad = 18.75\ foot\ kipf$

$M0 := T L / (4\ d) \quad = 22.72\ foot\ kipf$

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$$\beta := \frac{4 \sinh(\lambda L/2)^2}{\lambda L \sinh(\lambda L)} = 585.03 \times 10^{-3}$$

Torsional moment $MT := \beta M0 \quad = 13.29\ foot\ kipf$

Check Torsional Capacity (AISC 360-10 H3.3 & Philip page 100)

Maximum combined normal stress at the load point

$fbx := Mx / Sx + 2\,MT / Sy = 21.96\ \frac{kipf}{inch^2}$

Safety factor for compression

$\Omega := 1.67$:

$Fnx := Fy / \Omega = 29.94\ ksi$

If this is less than 1, then design is satifactory

$fbx / Fnx = 0.733$

Check Combined Compression and Bending Capacity (AISC 360-10 H1)

$$Mrx := \left(\frac{Mx}{Sx} + \frac{2\,MT}{Sy} \right) Sx = 109.78\ foot\,kipf$$

Effective length factor

$K := 0.85$:

Elastic bucking stress

$$Fe := \frac{\pi^2 E}{(K\,L / rx)^2} = 233.50\ ksi$$

Critical stress

$$Fcr := 0.658^{\frac{Fy}{Fe}}\ Fy = 45.71\ ksi$$

$$Pn := Fcr\,A \qquad = 23.24 \times 10^6\ poundal$$

Allowable axial strength

(this is greater than 3/4 Pr, so it is satisfactory)

$$Pc := Pn / \Omega \qquad = 1.92 \times 10^6\ N$$

Available flexural strength (Chapter F AISC 360-10)

$$Mn := \min(Fy\,Zx, Fy\,Sx) = 250.00\ foot\,kipf$$

$$Mcx := Mn / \Omega = 149.70\ foot\,kipf$$

This is greater than Mrx, so it is satisfactory

$$Mcy := \frac{Mn}{\Omega} = 149.70\ foot\,kipf$$

This should be below 1 for a satisfactory design

$$\frac{P}{Pc} + \frac{8}{9} \cdot \left(\frac{Mrx}{Mcx} + \frac{My}{Mcy} \right) = 985.16 \times 10^{-3}$$

Determine Deflections

Max twist angle (Lin, p100 eq4) in degrees

$$\phi := \frac{T}{2\,G\,J\,T\,\lambda} \cdot \left(\frac{\lambda \cdot L}{2} - \frac{2 \cdot \sinh\left(\frac{\lambda \cdot L}{2}\right)}{\sinh(\lambda \cdot L)} \right) \cdot \sinh\left(\frac{\lambda \cdot L}{2}\right) = 230.44 \times 10^{-3}$$

$$I3 := Ix \sin\left(\frac{(90-\phi)\pi}{180}\right)^2 + Iy \cos\left(\frac{(90-\phi)\pi}{180}\right)^2 = 303.00\ in^4$$

$$I4 := Ix \cos\left(\frac{(90-\phi)\pi}{180}\right)^2 + Iy \sin\left(\frac{(90-\phi)\pi}{180}\right)^2 = 103.00\ in^4$$

Vertical deflection at the middle $\Delta vert := \frac{5\,w\,L^4}{384\,E\,I3} = 0.15\ in$

Horizontal deflection at the middle $\Delta horiz := \frac{F\,L^3}{48\,E\,I4} = 0.20\ in$