Worst Case Circuit Analysis with Monte Carlo Simulation

Introduction

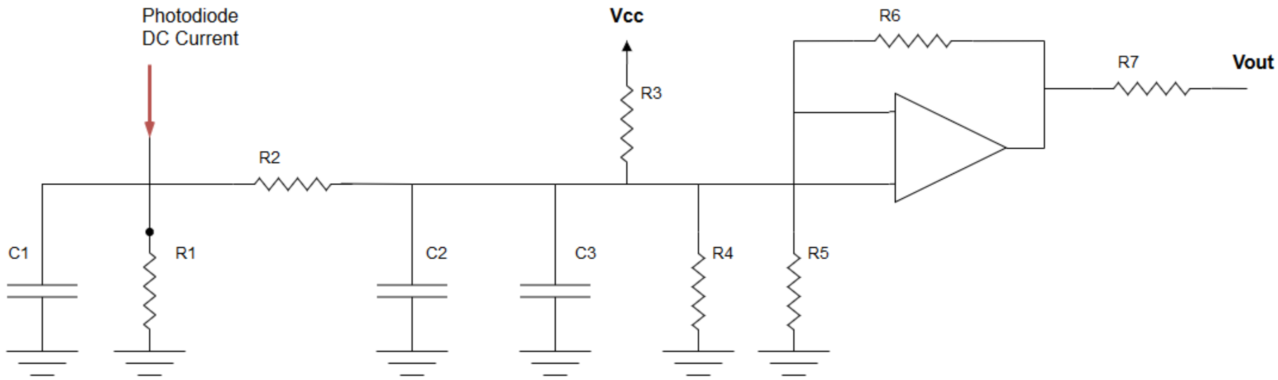
Electrical components (such as resistors and capacitors) are manufactured in large quantities. Inconsistencies in the raw materials and the manufacturing process means that component parameters have a statistical distribution. That is, the resistance of a batch of resistors might be described by a normal distribution.

Given the number of components in a circuit and the distribution of their parameters, the circuit may not perform as specified. This is a risk that must be identified, managed and mitigated early in the design process.

Worst Case Circuit Analysis (WCCA) is a set of techniques used to analyze how variations in parameters influence circuit performance. One approach is Monte Carlo analysis, in which parameters are randomly selected from a distribution, and the circuit simulated, anywhere from 1000 to 100000 times.

Given the results, circuits can be redesigned to minimize failures due to parameter variations (or an initially overdesigned circuit could be made cheaper to manufacture with less costly components that have a broader parameter distribution).

This application analyses the performance of this circuit using a Monte Carlo approach.



Light hits a photodiode, with an op-amp generating a linearly-proportional voltage from the current from the photodiode. Each parameter in the circuit is described by a normal probability distribution with a mean and variance given in the table below.

Component	Mean Value	Variance	Unit
R1	900	180	Ω
R2	67500	1350	Ω
R3	2050000	20500	Ω
R4	89200	1338	Ω
R5	90000	1350	Ω
R6	87000	435	Ω
R7	1.02	0.0714	Ω
Vcc	3	0.03	V
P	4.8×10^{-4}	0.000024	W

Calculations

```
> restart:
  with(Statistics):
  with(plottools):
  with(plots):

> Vout := proc(R1, R2, R3, R4, R5, R6, R, Vcc, P)
     return (R * P * R1 * (1 / (1 / R4 + 1 / R3)) / ( R1 + R2 + 1
  / (1 / R4 + 1 / R3)) + Vcc / R3 * 1 / (1 / R4 + 1 / R3 + 1 / (R2
  + R1))) * (R5 + R6) / R5;
  end proc:
```

Generate 1000 random samples for each parameter

```
> R1   := Sample(Normal(9000,    180),      1000):
  R2   := Sample(Normal(67500,   1350),     1000):
  R3   := Sample(Normal(2050000, 20500),    1000):
  R4   := Sample(Normal(89200,   1338),     1000):
  R5   := Sample(Normal(90000,   1350),     1000):
  R6   := Sample(Normal(87000,   435),      1000):
  R    := Sample(Normal(1.02,    0.0714),   1000):
  Vcc  := Sample(Normal(3,       0.03),     1000):
  P    := Sample(Normal(0.00048, 0.000024), 1000):
```

v is a vector that contains Vout for each run of the Monte Carlo simulation

```
> v := Vout~(R1, R2, R3, R4, R5, R6, R, Vcc, P)
```

v := [4.70757136754652, 5.04169978060156, 4.29945276522189, 4.04106635693425, **(2.1)**
5.07237399332086, 5.12665668197897, 4.49572458117545, 4.46235550016348,
4.18091354107599, 4.63683665943569, 5.01166567852514, 4.69106696820664,
4.67024512875014, 4.00490319323811, 4.81030531105047, 4.34145805043157,
4.48653364794331, 5.18139990974166, 4.84968575511678, 5.38650793236049,
4.72225414365793, 4.17414093620148, 4.36620999714376, 4.40310553123495,
4.61071218585563, 4.41758529968276, 4.86754291363466, 5.08907449647755,
4.62000175985595, 4.20501262384144, 4.68895286444307, 5.44583993704561,
4.67328719978007, 5.33311250728345, 4.48836936693673, 3.83287396850750,
4.60464220211853, 4.56407624378083, 4.30818566870511, 3.76266004702259,
4.85538869054042, 4.56859553040621, 4.62656115896785, 4.02475111321728,
4.95869915539946, 4.44784717482648, 4.43866322592115, 5.73078305155543,
4.67982168279172, 4.62383474672793, 5.01656083157381, 5.10135957043441,
4.64564669440393, 4.72158744140885, 4.65923125836712, 4.85433729208476,
4.84045714212141, 4.71052691069032, 5.20653761992328, 4.66376350117680,
4.74056941087772, 4.74613886077936, 4.53727642494872, 4.74166243187386,
4.74854524328914, 4.77038936514896, 4.64900556395877, 4.46993068961872,
4.26757321991738, 4.09996244785666, 4.85540043505398, 4.20525554450941,
4.12672961050896, 4.82559363992962, 4.61887841313731, 4.06976149082660,
4.41455133715270, 4.56409317585005, 5.30423250592347, 4.56959382325416,
4.32186601822580, 5.21391271994067, 4.97225234491135, 4.94945446889095,
4.40363757463315, 4.07337539928911, 4.49816968018508, 4.85653970762846,
4.39996507337152, 4.65078636481225, 5.27807569892357, 4.56969068682801,
4.63858672572383, 5.14076437678530, 4.51957128757126, 5.45373352879232,
5.21938964490708, 4.11639432734211, 4.48106126072140, 4.36609337693519,...,
... 900 row vector entries not shown]

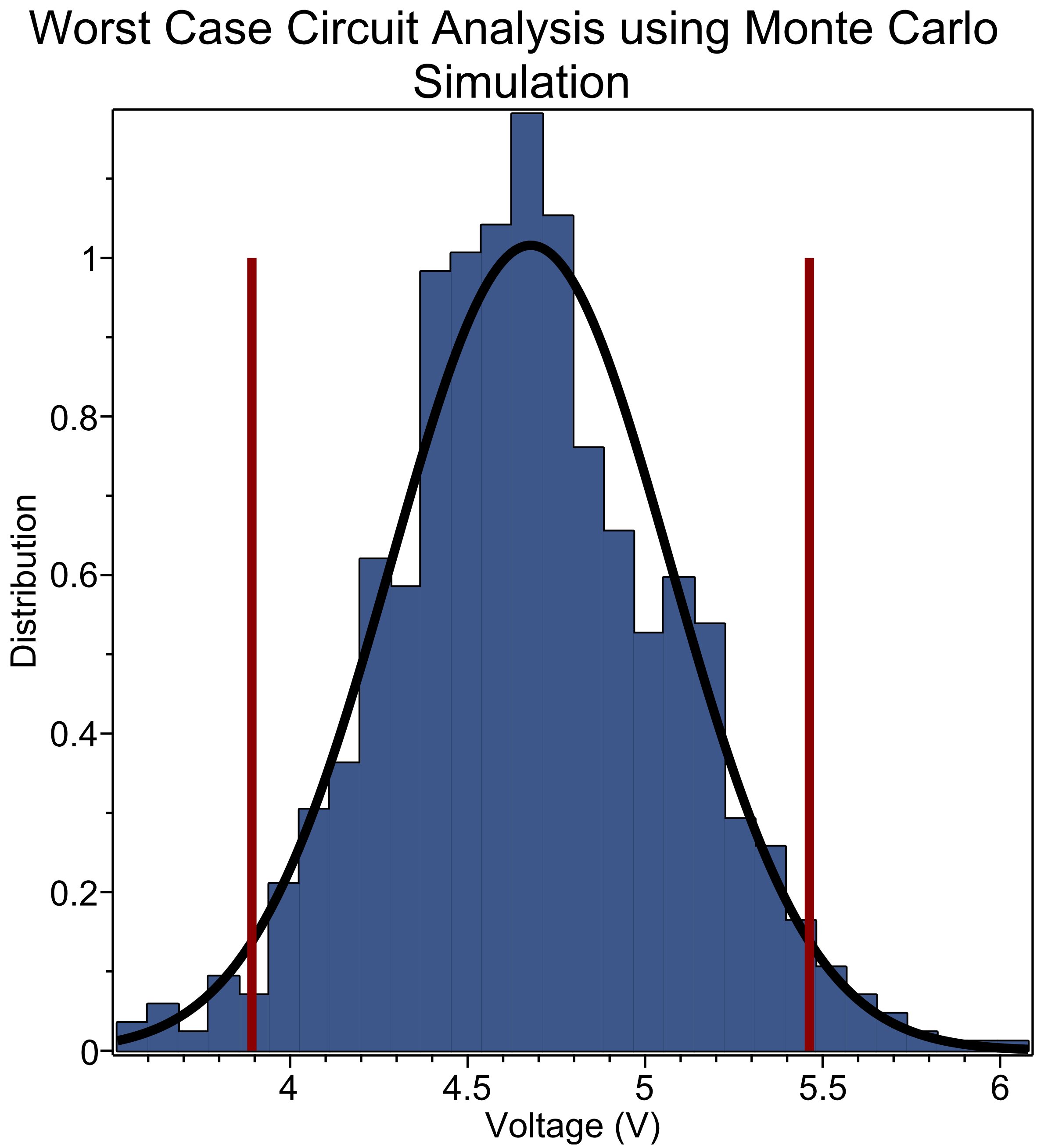
Mean and standard deviation of results

```
> mu    := Mean(v);
  sigma := StandardDeviation(v)
```

$$\mu := 4.67742307156047$$
$$\sigma := 0.392665397536949 \tag{2.2}$$

Plot a histogram with 2σ limits (95.44% of the observations lie between 2 standard deviations of the mean,)

```
> p1 := Histogram(v, labels = ["Voltage (V)","Distribution"],
  labeldirections = [horizontal, vertical], labelfont = [Arial],
  size = [800, 500], title = "Worst Case Circuit Analysis using
  Monte Carlo Simulation", titlefont = [Arial, 16], axesfont =
  [Arial]):
  p2 := DensityPlot(Normal(mu, sigma), range = min(v)..max(v),
  color = black, thickness = 4):
  p3 := line([mu - 2*sigma, 0],[mu - 2*sigma, 1], color =
  "DarkRed", thickness = 4), plottools:-line([mu + 2*sigma, 0],
  [mu + 2*sigma, 1], color = "DarkRed",thickness = 4):
  display(p1, p2, p3);
```


Worst Case Circuit Analysis using Monte Carlo Simulation
Distribution
0
0.2
0.4
0.6
0.8
1
4
4.5
5
5.5
6
Voltage (V)