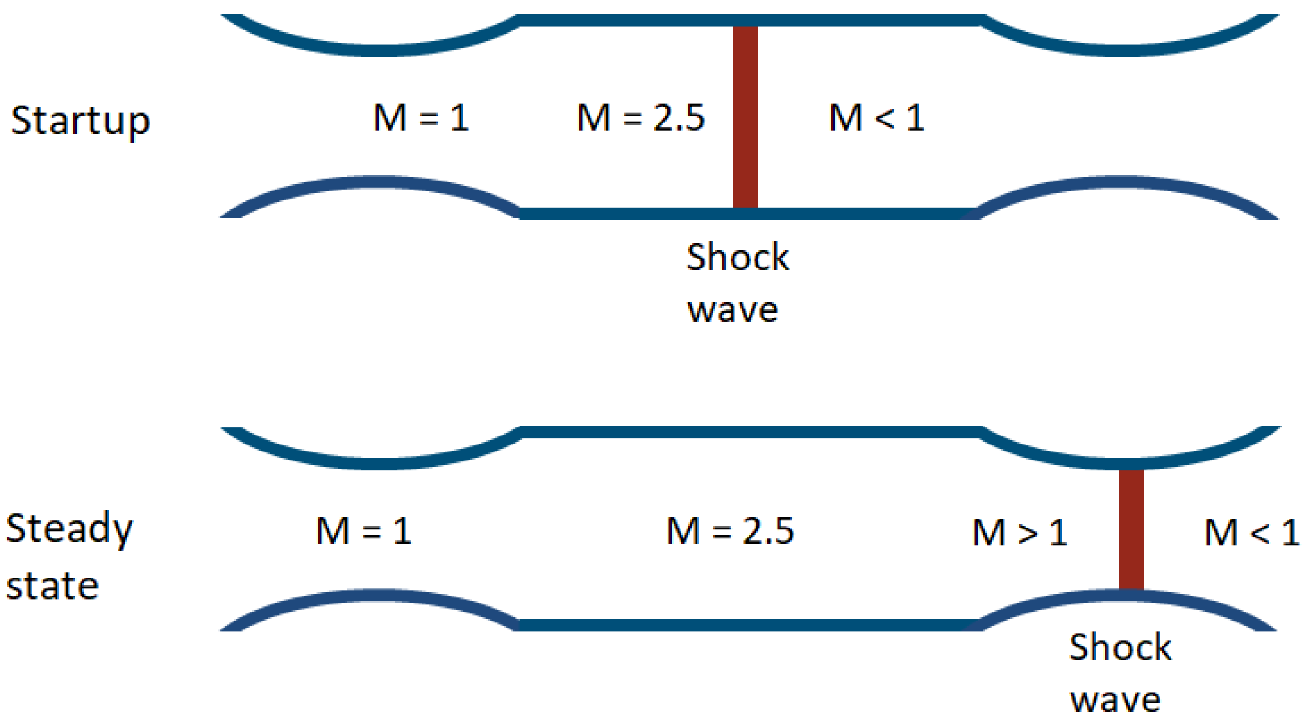
Compressor Power for a Supersonic Wind Tunnel at Steady- state and Start-up

Introduction

This application calculates the compressor power (at steady-state and at start-up) for a fixed geometry supersonic wind tunnel. The test section will operate at Mach 2.5, simulate an altitude of 21 km and has a circular cross-sectional area with a diameter of 25 cm. A supersonic fixed-area diffuser follows the test section.



A cooler between the compressor and the nozzle ensures that the air at the compressor inlet and in the test section have the same stagnation temperature.

The air entering the compressor and in the test section has the same stagnation temperature. The compressor is isentropic, and friction and boundary layer effects are not considered.

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> restart :
  with(Units[Simple]) :
  with(ThermophysicalData) :
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US Standard Atmosphere 1976

Parameters

Diameter of test section (circular)

> $D_{test} := 0.25\ \mathrm{m}$:

Mach number in test section

> $M_{test} := 2.5$:

Cross-sectional area of test section

> $A_{test} := \dfrac{\pi D_{test}^{2}}{4}$

$$4.91 \times 10^{-2}\ \mathrm{m}^2 \tag{2.1}$$

Design altitude

> $height := 21\ \mathrm{km}$

$$height := 21\ \mathrm{km} \tag{2.2}$$

Flight conditions given the altitude

> $T_{test}, P_{test}, \rho_{test}, a_{test}, \mu_{test} := \mathrm{Atmosphere}(height, useunits = true)$

$$2.18 \times 10^{2}\ \mathrm{K}, 4.68 \times 10^{3}\ \mathrm{Pa}, 7.49 \times 10^{-2}\ \frac{\mathrm{kg}}{\mathrm{m}^3}, 2.96 \times 10^{2}\ \frac{\mathrm{m}}{\mathrm{s}}, 1.43 \times 10^{-5}\ \mathrm{Pa\,s} \tag{2.3}$$

Specific heat ratio

> $k := 1.4$:

Specific heat at constant pressure

> $Cp := \mathrm{Property}(C, air, pressure = P_{test}, temperature = T_{test})$

$$1.003\ \frac{\mathrm{kJ}}{\mathrm{kg\,K}} \tag{2.4}$$

Stagnation Properties

At steady-state, the optimum operating condition occurs with a normal shock (perpendicular to the flow direction) at the diffuser throat and $A_{throat} < A_{diffuser}$

Ratio of the local static temperature to the stagnation temperature (isentropic flow)

$$> TR_{isen} := \left(1 + \frac{k-1}{2} M_{test}^{2}\right)^{-1}$$

$$0.444 \tag{3.1}$$

Stagnation temperature

$$> T_{test\_stag} := \frac{T_{test}}{TR_{isen}}$$

$$489.71 \text{ K} \tag{3.2}$$

Given that the upstream flow is isentropic, M_{test} is the upstream Mach number

Total pressure ratio $PR_{test\_stag} = Pt_{diffuser} / Pt_{nozzle}$ (normal shock wave)

$$> PR_{test\_stag} := \left(\frac{(k+1) \cdot M_{test}^{2}}{(k-1) \cdot M_{test}^{2} + 2}\right)^{\frac{k}{k-1}} \cdot \left(\frac{k+1}{2 \cdot k \cdot M_{test}^{2} - (k-1)}\right)^{\frac{1}{k-1}}$$

$$0.50 \tag{3.3}$$

Due to the conservation of mass, the area ratio $AR_{shock} = A_{nozzle} / A_{diffuser}$ is equal to the pressure ratio (isentropic flow)

$$> AR_{shock} := PR_{test\_stag} :$$

Area ratio $AR_{isen} = A / A^*$ (A* occurs when flow is choked and the Mach number is equal to 1)

$$> AR_{isen} := \left(\frac{k+1}{2}\right)^{-\frac{k+1}{2(k-1)}} \frac{\left(1 + \frac{k-1}{2} M_{test}^{2}\right)^{\frac{k+1}{2(k-1)}}}{M_{test}}$$

$$2.637 \tag{3.4}$$

Diffuser area

$$> A_{diffuser} := \frac{A_{test}}{AR_{shock} AR_{isen}}$$

$$3.73 \times 10^{-2} \text{ m}^{2} \tag{3.5}$$

Since $A_{diffuser} < A_{test}$, the Mach number converges to 1

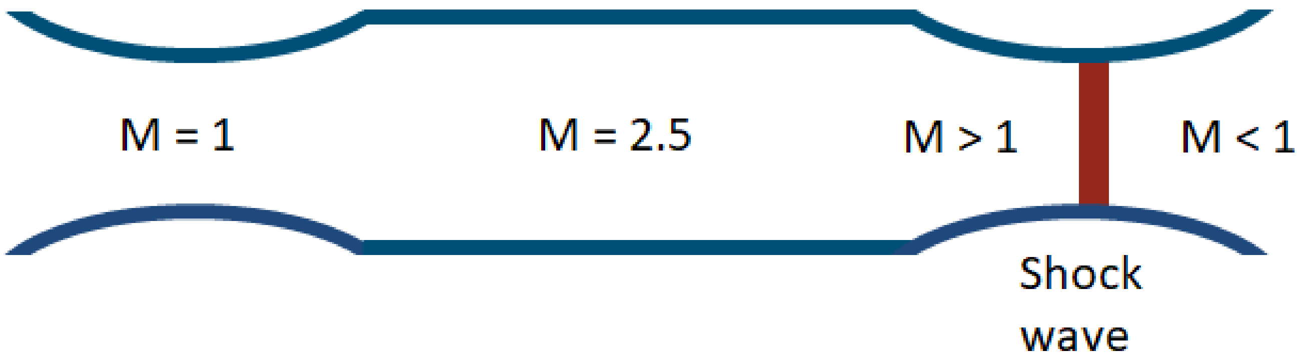
Mach number upstream of shock (isentropic flow)

$$> M_{diff\_up\_shock} := fsolve\left(\frac{1}{AR_{shock}} = \frac{\left(\frac{k+1}{2}\right)^{-\frac{k+1}{2(k-1)}}\left(1+\frac{k-1}{2}M^2\right)^{\frac{k+1}{2(k-1)}}}{M}, M = M_{test}\right)$$

$$2.20 \tag{3.6}$$

Compressor Power at Steady-State

At steady-state, the shock wave is in the diffuser throat



Total pressure ratio through the shock wave (normal shock wave)

$$> PR_0 := \left(\frac{(k+1)\,M_{diff\_up\_shock}^{\,2}}{(k-1)\,M_{diff\_up\_shock}^{\,2}+2}\right)^{\frac{k}{k-1}}\left(\frac{k+1}{2\,k\,M_{diff\_up\_shock}^{\,2}-(k-1)}\right)^{\frac{1}{k-1}}$$

$$0.63 \tag{4.1}$$

Temperature difference across the isentropic compressor (bearing in mind that the

$$> \Delta T := T_{test\_stag}\left(\left(\frac{1}{PR_0}\right)^{\frac{k-1}{k}}-1\right)$$

$$69.51\ K \tag{4.2}$$

Mass flowrate through the test section at steady state

$$> m := \rho_{test}\,A_{test}\,M_{test}\,a_{test}$$

$$2.72\ \frac{kg}{s} \tag{4.3}$$

Reduction in the stagnation pressure must be compensated for by the compressor

Specify work of compressor

> $W_{steady} := Cp\,\Delta T$

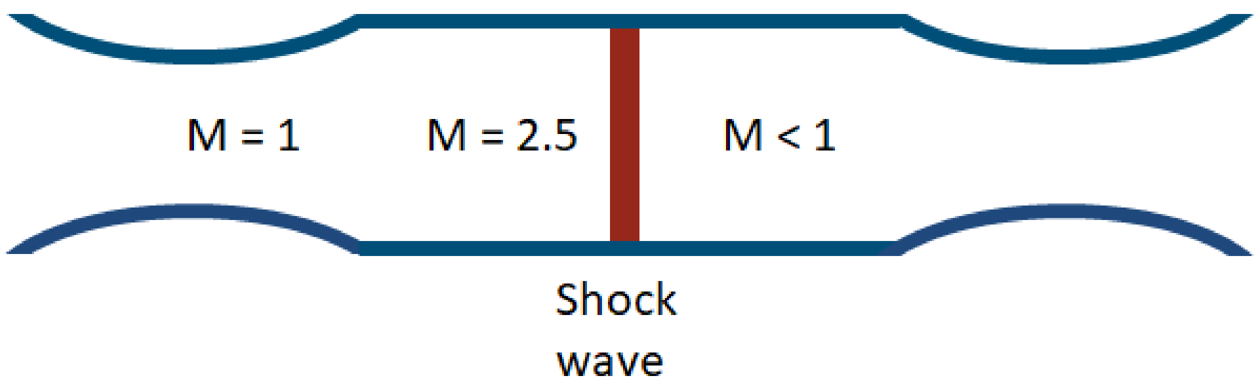
$$69.70\ \frac{kJ}{kg} \tag{4.4}$$

Power required by the compressor at steady-state

> $P_{steady} := W_{steady}\, m$

$$189.41\ kW \tag{4.5}$$

Compressor Power at Startup



At startup, the shock wave is in the test section (normal shock wave)

> $PR_{stag\_start} := \left(\frac{(k+1)\, M_{test}^{2}}{(k-1)\, M_{test}^{2}+2}\right)^{\frac{k}{k-1}} \left(\frac{k+1}{2\,k\,M_{test}^{2}-(k-1)}\right)^{\frac{1}{k-1}}$

$$0.50 \tag{5.1}$$

Specific work of compressor

> $W_{start} := Cp\, T_{test\_stag} \left(\left(\frac{1}{PR_{stag\_start}}\right)^{\frac{k-1}{k}} - 1\right)$

$$107.88\ \frac{kJ}{kg} \tag{5.2}$$

Power at startup

> $P_{start} := W_{start}\, m$

$$293.16\ kW \tag{5.3}$$