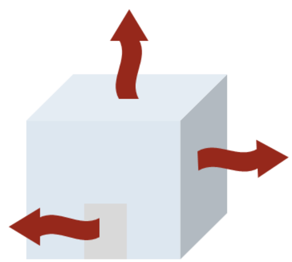
Cost of Heating a Home with Natural Gas

Introduction

This application calculates the cost of heating a house with natural gas, assuming

- heat losses through four side walls (each with windows), a roof, and air exchanges with the environment
- and heat gains through a furnace.



Historical heating degree day data for Kitchener, Ontario, will be used.

Only the direct cost of the natural gas is computed; no other costs, such as delivery or taxes, are included.

```
> restart :
  with( ThermophysicalData ) :
  with( TimeSeriesAnalysis ) :
  with( ColorTools ) :
  with( Units[Standard] ) :
```

Parameters

Humid Air

Properties of humid air at 18°C and 1 atmosphere, and a relative humidity of 50%

Density

$$> \rho := \frac{1}{\mathrm{Property}(V, \mathrm{Tdb} = 18\ \mathrm{degC}, \mathrm{pressure} = 1\ \mathrm{atm}, R = 0.5, \mathrm{HumidAir})}$$

(1.1.1)

$$1.20\ \frac{kg}{m^3} \tag{1.1.1}$$

Specific heat capacity

> airCapacity := Property(C, Tdb = 16 degC, P = 1 atm , R = 0.5, HumidAir)

$$1010.92\ \frac{J}{kg\,K} \tag{1.1.2}$$

Natural Gas and Furnace

Cost of natural gas per unit volume at standard conditions

> costPerVolume := $0.0972\ m^{-3}$:

Energy content of natural gas

> energyContentPerVolume := $0.0373\ GJ\,m^{-3}$:

Wall and Ceiling

Area of side 1, side 2, side 3, side 4 and ceiling

> $A_{wallceiling} := \begin{bmatrix} 60 \\ 60 \\ 60 \\ 60 \\ 100 \end{bmatrix} m^2$:

Depth of insulation in side 1, side 2, side 3, side 4 and ceiling

> $insulationDepth_{wallceiling} := \begin{bmatrix} 9 \\ 9 \\ 9 \\ 9 \\ 15 \end{bmatrix} cm$:

R value of insulation (typical value for fiberglass batts)

> $Rp := 22\ \frac{K\,m}{W}$:

Windows

Area and R values of windows on side 1, side 2, side 3, side 4, and ceiling (note that there are no windows on one side and on the ceiling)

$$> A_{window} := \begin{bmatrix} 5.4 \\ 12 \\ 2.4 \\ 0 \\ 0 \end{bmatrix} m^2 :$$

$$> R_{window} := \begin{bmatrix} 0.35 \\ 0.35 \\ 0.35 \\ 0.35 \\ 10^8 \end{bmatrix} \frac{m^2 \cdot K}{W} :$$

Miscellaneous

Volume of building interior

$> \text{volumeBuilding} := 400\ m^3 :$

Furnace efficiency

$> \text{furnanceEfficiency} := 0.95 :$

Number of air exchanges per hour

$> \text{airExchanges} := 0.8\ \text{hour}^{-1} :$

Historical Heating Degree Days for Kitchener, Ontario

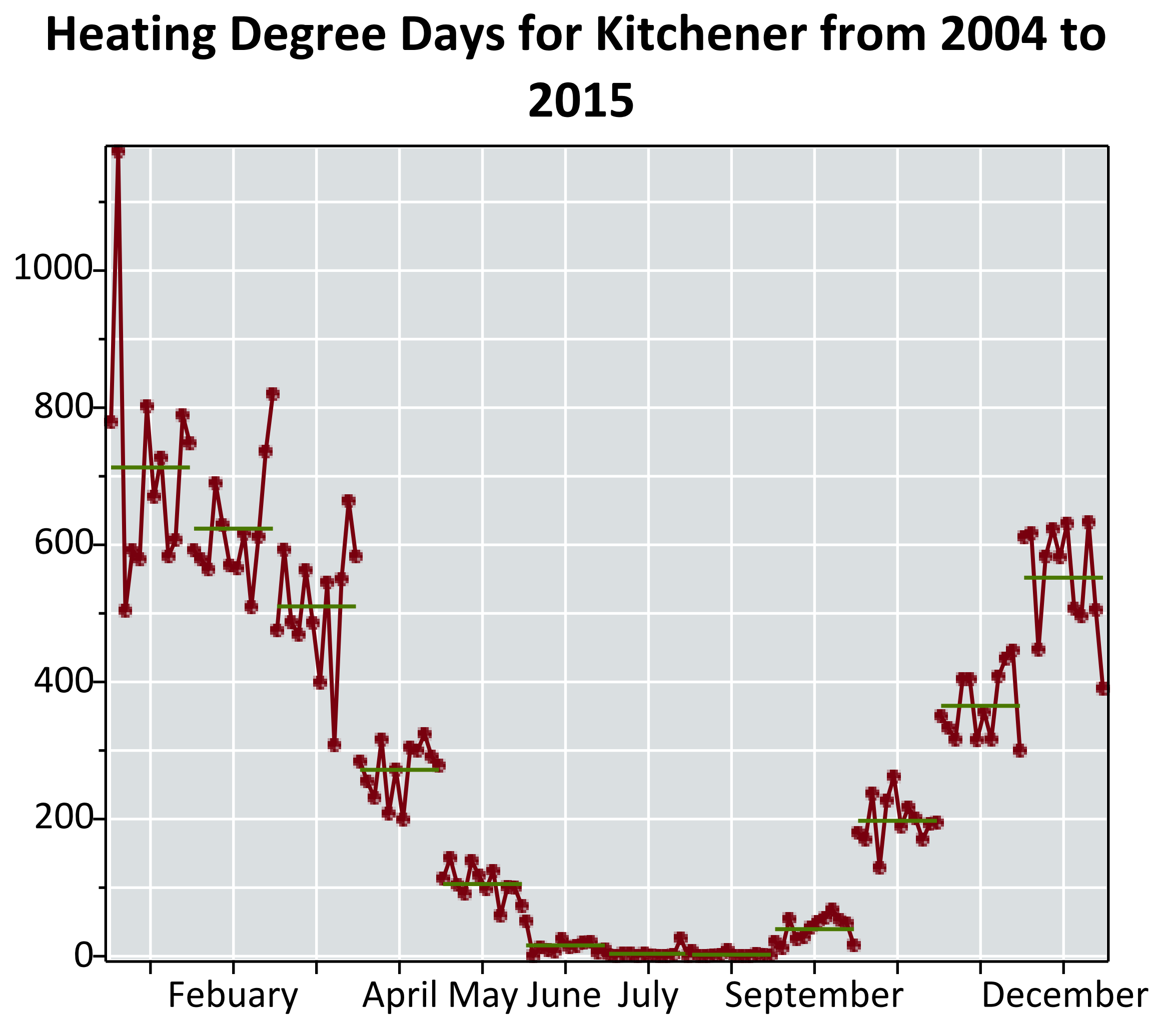
Import and visualize HDD data for weather station code YKF and a base temperature of 16°C.

```
> data := ImportMatrix("this://HDD YKF Jan 2004- Dec 2015 Monthly.csv")[2 .., 2]:
> ts := TimeSeries(data, frequency = monthly, startdate = "2004-01-01", period = 12)
```

$$ts := \begin{bmatrix} \textit{Time series} \\ \textit{data set} \\ \textit{144 rows of data:} \\ \textit{2004-01-01 - 2015-12-01} \end{bmatrix} \qquad \textbf{(1.6.1)}$$

```
> SeasonalSubseriesPlot(ts, seasonnames = ["January", "Febuary", "March", "April", "May",
    "June", "July", "August", "September", "October", "November", "December"], size
    = [1000, 400], symbol = solidcircle, axesfont = [Calibri], title
    = "Heating Degree Days for Kitchener from 2004 to 2015", titlefont = [Calibri, 16,
    bold], background = Color("RGB", [218/255, 223/255, 225/255]), axis = [gridlines
```

= [color = Color("RGB", [1, 1, 1])]])



Mean HDD for January in Kitchener, Ontario

> HDD := 713 degC·day :

Cost of Natural Gas

Cost per unit energy content of natural gas

> $\text{costPerEnergy} := \dfrac{\text{costPerVolume}}{\text{energyContentPerVolume}}$

$$2.61 \times 10^{-3} \frac{1}{\text{MJ}} \quad \textbf{(2.1)}$$

Effective cost of per energy content of natural gas, given the furnace efficiency

> $\text{costPerEnergyEffective} := \dfrac{\text{costPerEnergy}}{\text{furnanceEfficiency}}$

(2.2)

$$2.74 \times 10^{-3} \frac{1}{MJ} \quad (2.2)$$

Energy Losses in One Month Through Various Paths

Walls and Ceilings

R values of side 1, side 2, side 3, side 4 and ceiling

> $R_{wallceiling} := Rp\,insulationDepth_{wallceiling}$

$$R_{wallceiling} := \begin{bmatrix} 198 \frac{K\,m}{W}\,cm \\ 198 \frac{K\,m}{W}\,cm \\ 198 \frac{K\,m}{W}\,cm \\ 198 \frac{K\,m}{W}\,cm \\ 330 \frac{K\,m}{W}\,cm \end{bmatrix} \quad (3.1.1)$$

> $Q_{wallceiling} := HDD \cdot add\left(\frac{A_{wallceiling}[i] - A_{window}[i]}{R_{wallceiling}[i]}, i = 1..5\right)$

$$8.72 \times 10^{3}\ MJ \quad (3.1.2)$$

Windows

> $Q_{window} := HDD \cdot add\left(\frac{A_{window}[i]}{R_{window}[i]}, i = 1..5\right)$

$$3.48 \times 10^{3}\ MJ \quad (3.2.1)$$

Air Exchanges

Heat capacity of air per unit volume

> $airCapacityPerVolume := airCapacity \cdot \rho$

$$1.21 \times 10^{3} \frac{J}{K\,m^{3}} \quad (3.3.1)$$

> $Q_{exchange} := HDD \cdot volumeBuilding \cdot airExchanges \cdot airCapacityPerVolume$

$$6.65 \times 10^{3}\ MJ \quad (3.3.2)$$

Total Energy Loss in One Month

> $Q_{total} := Q_{wallceiling} + Q_{window} + Q_{exchange}$

$$1.88 \times 10^4 \text{ MJ} \quad (3.4.1)$$

Cost Calculation

Total cost of natural gas used in heating home for one month

> $cost := Q_{total} \cdot costPerEnergyEffective$

$$cost := 51.7022471161316 \quad (4.1)$$

>