Maple 2024 includes a number of improvements to support teaching and learning of mathematics and science.

Step-by-Step Solutions

Maple 2024 improves the existing suite of commands for showing step-by-step solutions to standard math problems. It also adds some new methods as follows:

Partial Fraction Decomposition

Partial Fraction Decomposition is a method used in algebra and calculus to simplify complex fractions or rational expressions into a sum of simpler fractions.

Maple can show you the steps required to perform partial fraction decomposition with the new command <u>PartialFractionSteps</u>.

with(Student:-Basics) :

$$PartialFractionSteps\left(-\frac{4x^{2}+4x}{(2x^{2}+8x+6)\cdot(x+2)}\right) -\frac{4x^{2}+4x}{(2x^{2}+8x+6)(x+2)}$$

• First, factor the numerator and denominator. If we can simplify by dividing terms, we do so.

$$\frac{2x}{(x+3)(x+2)}$$

• Use the factors to write an equation for the partial fraction decomposition

$$\frac{2x}{(x+3)(x+2)} = \frac{A}{x+2} + \frac{B}{x+3}$$

• Multiply by the LHS's denominator.

$$-2x = \left(\frac{A}{x+2} + \frac{B}{x+3}\right)(x+3)(x+2)$$

• Expand everything out.

-2x = Ax + Bx + 3A + 2B

• Equate the coefficients to create equations

$$[0 = 3A + 2B, -2 = A + B]$$

• Solve the equations.

$$[A=4, B=-6]$$

• Use these values in the partial fraction decomposition and simplify. The final partial fraction decomposition is

$$-\frac{6}{x+3} + \frac{4}{x+2}$$

Solve Steps

Maple 2024 improves the steps given by the <u>SolveSteps</u> command.

The following example recognizes $\ln(x^2)$ as a common factor and goes into greater detail when applying log rules.

with(Student:-Basics) :

SolveSteps
$$\left(\ln(x^2) = 2 \cdot \left(\frac{\ln(x^2)}{2} \right)^2 \right);$$

Let's solve

$$\ln(x^2) = \frac{\ln(x^2)^2}{2}$$

• Simplify

$$\ln(x^2) - \frac{\ln(x^2)^2}{2} = 0$$

• Factor

$$\ln(x^2) - \frac{\ln(x^2)^2}{2} = 0$$

• Remove rationals and common factor

$$-\frac{1}{2}\cdot\ln(x^2)\cdot(-2+\ln(x^2))$$

- The 1st factor is $\ln(x^2)$ which implies $\ln(x^2) = 0$ is a solution $\ln(x^2) = 0$
- Set 2nd factor $-2 + \ln(x^2)$ to 0 to solve $-2 + \ln(x^2) = 0$
- Solution of $-2 + \ln(x^2) = 0$ $\ln(x^2) = 2$
- Solution

$$\ln(x^2) = (0,2)$$

Consider 1st solution

$$\ln(x^2) = 0$$

- Raise both sides to a power of e $e^{\ln(x^2)} = e^0$
- Log rule: $e^{\ln(X)} = X$ $x^2 = e^0$
- Simplify

 $x^2 = 1$

• Take Square root of both sides

$$x = \pm \sqrt{1}$$

• Consider 2nd solution

$$\ln(x^2)=2$$

• Raise both sides to a power of e

$$e^{\ln(x^2)} = e^2$$

• Log rule: $e^{\ln(X)} = X$

$$x^2 = e^2$$

• Take Square root of both sides

 $x = \pm \sqrt{e^2}$

Check if x = -1 satisfies domain requirements

Domain requirement from $\ln(x^2)$, cannot take log of a negative number

 $0 \le x^2$

Substitute x = -1 into x^2 and evaluate

 $(-1)^2$

Evaluate exponents

1

Domain requirement met

 $0 \le 1$

Therefore the domain requirement is satisfied.

 \checkmark

Substitute solution into equation and check if left-side = right-side

0 = 0

left-side = right-side

• x = -1 satisfies all the domain requirements and left-side = right-side, it is a solution \checkmark

Check if x = 1 satisfies domain requirements

Domain requirement from $\ln(x^2)$, cannot take log of a negative number $0 \le x^2$

Substitute x = 1 into x^2 and evaluate

 1^2

Domain requirement met

 $0 \le 1$

Therefore the domain requirement is satisfied.

 \checkmark

Substitute solution into equation and check if left-side = right-side

0 = 0

left-side = right-side

0 = 0

x = 1 satisfies all the domain requirements and left-side = right-side, it is a solution \checkmark

Check if $x = \sqrt{e^2}$ satisfies domain requirements

Domain requirement from $ln(x^2)$, cannot take log of a negative number

 $0 \le x^2$

Substitute
$$x = \sqrt{e^2}$$
 into x^2 and evaluate

 $\left(\sqrt{e^2}\right)^2$

Evaluate exponents

 e^2

Evaluate

7.389056099

Domain requirement met

 $0 \le 7.389056099$

Therefore the domain requirement is satisfied.

 \checkmark

- Substitute solution into equation and check if left-side = right-side
 2 = 2
- left-side = right-side

2 = 2

• $x = \sqrt{e^2}$ satisfies all the domain requirements and left-side = right-side, it is a solution

 \checkmark

Check if $x = -\sqrt{e^2}$ satisfies domain requirements

Domain requirement from $\ln(x^2)$, cannot take log of a negative number $0 \le x^2$

Substitute $x = -\sqrt{e^2}$ into x^2 and evaluate

 $\left(-\sqrt{e^2}\right)^2$

Evaluate exponents

e²

Evaluate

7.389056099

Domain requirement met

 $0 \le 7.389056099$

Therefore the domain requirement is satisfied.

 \checkmark

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Substitute solution into equation and check if left-side = right-side

2 = 2

• left-side = right-side

2 = 2

- $x = -\sqrt{e^2}$ satisfies all the domain requirements and left-side = right-side, it is a solution $\sqrt{}$
- Exact solutions

 $x = \{-1, 1, \sqrt{e^2}, -\sqrt{e^2}\}$

Approximate solutions
 x = { -2.718281828, -1., 1., 2.718281828 }

In the example below, the step-by-step solution uses rules to handle logs of constants that are perfect powers. Note that we use the <u>inert-form</u> name %*log* instead of *log* in order to prevent evaluation.

with(Student:-Basics) :

$$SolveSteps\left(\frac{\% log[11](25)}{\% log[11](5)} = 6x\right);$$

Let's solve

$$\frac{\log_{11}(25)}{\log_{11}(5)} = 6 \cdot x$$

Rearrange expression

$$6 \cdot x = \frac{\log_{11}(25)}{\log_{11}(5)}$$

- Rewrite $\log_{11}(25)$ as $\log_{11}(5^2)$, then apply the log rule $\log_{11}(5^2) = 2\log_{11}(5)$ 6x = 2
 - Divide both sides by 6

$$\frac{6 \cdot x}{6} = \frac{2}{6}$$

• Simplify

$$x = \frac{1}{3}$$

When solving systems of equations such as the following, it is important to select the right variable to start with. SolveSteps analyzes the system and chooses an appropriate variable to start with, thus avoiding more complicated steps.

SolveSteps(
$$[y = -x^2 + 6x - 10, y = 2x^2 + 6x + 5]$$
);

Let's solve

$$[y = (-1) \cdot x^{2} + 6 \cdot x - 10, y = 2 \cdot x^{2} + 6 \cdot x + 5]$$

• E1: Pick the 2nd equation which is already solved for y

 $y = 2 \cdot x^2 + 6 \cdot x + 5$

• Pick the 1st equation to solve for *x*

$$y = (-1) \cdot x^2 + 6 \cdot x - 10$$

• Substitute the value of $y = 2x^2 + 6x + 5$ into the 1st equation of the system $2x^2 + 6x + 5 = (-1) \cdot x^2 + 6 \cdot x - 10$

Isolate for *x*

Add $x^2 - 6x + 10$ to both sides $2x^2 + 6x + 5 + (x^2 - 6x + 10) = -x^2 + 6x - 10 + (x^2 - 6x + 10)$ Simplify

Simplify

$$3x^2 + 15 = 0$$

Subtract 15 from both sides

 $3 \cdot x^2 + 15 - 15 = 0 - 15$

Simplify

 $3 \cdot x^2 = -15$

Divide both sides by 3

$$\frac{3 \cdot x^2}{3} = \frac{-15}{3}$$

Simplify

$$x^2 = -5$$

Take Square root of both sides

$$x = \pm \sqrt{-5}$$

No real solutions

x = imaginary

Maple 2024 can now give steps in more situations involving expressions with absolute values. For example,

with(Student:-Basics):

SolveSteps ($|2x + 3| \ge 2$);

Let's solve

 $2 \le |2 \cdot x + 3|$

Solve for x to find points to test for intervals

 $2 = |2 \cdot x + 3|$

 To solve, we must drop the absolute values. To do this, we must determine the intervals where the expression within the absolute value becomes positive or negative

$$2x + 3 < 0 \quad \text{when} \quad x \in \left(-\infty, -\frac{3}{2}\right)$$
$$2x + 3 \ge 0 \quad \text{when} \quad x \in \left[-\frac{3}{2}, \infty\right)$$

Thus our intervals are:

$$\left(-\infty,-\frac{3}{2}\right),\left[-\frac{3}{2},\infty\right)\right]$$

- Examine absolute values with $x \in \left(-\infty, -\frac{3}{2}\right)$
 - Determine whether the inside of the absolute value will be positive or negative

 $\left[\begin{array}{c}2x+3<0\end{array}\right]$

• Drop the absolute values and multiply the expressions that would be negative by -1

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|2x+3| = -2x-3

 \circ $\;$ Sub the new expressions in where the absolute values used to be

2 = -2x - 3

• Solve the new equality

$$x = -\frac{5}{2}$$

• Since $-\frac{5}{2} \in \left(-\infty, -\frac{3}{2}\right)$ we get that this is a solution
 $x = -\frac{5}{2}$

• Examine absolute values with $x \in \left[-\frac{3}{2}, \infty\right]$

 $\circ~$ Determine whether the inside of the absolute value will be positive or negative

 $\left[\begin{array}{c}2x+3\geq0\end{array}\right]$

· Drop the absolute values and multiply the expressions that would be negative by -1

 $\begin{bmatrix} |2x+3| = -2x-3 \end{bmatrix}$

Sub the new expressions in where the absolute values used to be

2 = -2x - 3

Solve the new equality

$$x = -\frac{5}{2}$$

• Since $-\frac{5}{2} \in \left(-\infty, -\frac{3}{2}\right)$ we get that this is a solution

$$x = -\frac{5}{2}$$

• Examine absolute values with $x \in \left[-\frac{3}{2}, \infty\right)$

- Determine whether the inside of the absolute value will be positive or negative
 [2x + 3 ≥ 0]
- · Drop the absolute values and multiply the expressions that would be negative by -1

|2x+3| = 2x+3

- Sub the new expressions in where the absolute values used to be 2 = 2x + 3
- · Solve the new equality

$$x = -\frac{1}{2}$$

• Since $-\frac{1}{2} \in \left[-\frac{3}{2}, \infty\right]$ we get that this is a solution

$$x = -\frac{1}{2}$$

Solution

$$x = \left(-\frac{5}{2}, -\frac{1}{2}\right)$$

Use the solutions to the equality as points to test for intervals

$$\left[-\frac{5}{2},-\frac{1}{2}\right]$$

· Set up a table using the solutions as boundaries and find test points that are on either side

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-5/2 -1/2
-3 -1 0
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Sub each test point into the expression for x

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• Sub x = -3 into 2 \le |2x + 3|

2 \le 3

true

• Sub x = -1 into 2 \le |2x + 3|

2 \le 1

false

• Sub x = 0 into 2 \le |2x + 3|

2 \le 3

true
```

Observe where the inequality holds true, these areas make up the intervals



Plotted solution



Solution

$$\left\{x \le -\frac{5}{2}\right\}, \left\{-\frac{1}{2} \le x\right\}$$

Check My Work: Help with Math Problems

Building on the suite of <u>SolvePractice</u> / <u>SolveFeedback</u> commands, Maple 2024 introduces new commands to help identify where mistakes occur in worked problems relating to <u>factoring</u>, <u>simplification</u>, and <u>limits</u>.

Check My Work

 Grading:-FactorPractice($2 \cdot x^3 - 8 \cdot x$)

 Factor

 Please fill in your steps line by line.

 'Click on the button to check your work.

 $2x^3 - 8x$
 $2 \cdot x \cdot (x^2 - 4)$

 Ok

 $2 \cdot x \cdot (x - 2) \cdot (x + 2)$

 Good job, you have found the correct factorization

 > New Math Entry Box

Grading:-SimplifyPractice $(\cos(x)^3 + \cos(x)\sin(x)^2)$

Simplify

Check My Work

Please fill in your steps line by line. Click on the button to check your work.

Grading:-LimitPractice $\left(\begin{array}{c} Limit \left(\begin{array}{c} \frac{\sin(x)}{x}, x = 0 \end{array} \right) \right);$

Find this limit

Check My Work

Please fill in your steps line by line. Click on the button to check your work.



New Math Apps

Two Math Apps have been added to our collection.

New MathApp	Description
Tangent Planes	The <u>Tangent Planes</u> app explores surfaces defined by functions of two variables and shows the connection between a function's partial derivatives and the surface's tangent plane at a point.
Golden Angle	The <u>Golden Angle</u> app offers a visualization of the patterns formed by rotations of a given angle around a circle. The resulting pattern, using the golden angle itself, mimics patterns found in nature.